Modeling, Estimation, and Applications of Phase Noise in Wireless Communications: A Survey

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Abstract—Phase noise is inevitably present in wireless communications, arising from both hardware impairments and imperfect channel estimation, and has substantial implications on the integrity and efficiency of communication systems. However, phase noise brings more to wireless systems than just negative effects. For instance, phase noise can be used as a physical layer feature to authenticate received signals and as a source for random number generation. There is a surprising lack of a comprehensive overview of the state-of-the-art estimation methods and applications of phase noise in wireless communications. In this paper, we conduct the first comprehensive survey on the modeling, estimation, and applications of phase noise in wireless communications. We review the state-of-the-art phase-noise models, phase-noise estimation, and mitigation techniques via joint transmit-receive design, as well as applications of phase noise in wireless communications, while highlighting contributions and critical issues of each reviewed work. Finally, we provide an expansive set of research directions, accompanying challenges, and potential solutions related to phase noise that could shape the future of phase noise research in wireless communications.

Index Terms—Phase noise, modeling, estimation, application, mitigation.

I. INTRODUCTION

A. Background and Motivation

PHASE noise, a critical phenomenon in wireless communications, perturbs the stability of oscillators used in transmitters and receivers, introducing random fluctuations in the phase of the carrier wave. It arises from hardware impairments and/or imperfect channel estimation [1], [2], [3] and has substantial implications for the integrity and efficiency

of communication systems [4], [5], [6]. In an Orthogonal Frequency Division Multiplexing (OFDM) system, phase noise destroys the orthogonality between channels, increases the Bit Error Rate (BER), and decreases the Signal-to-Noise Ratio (SNR). In signal processing, phase noise complicates the extraction of signal parameters, directly affecting tasks such as modulation, demodulation, and frequency synthesis.

For wireless communications, the impact of phase noise is particularly severe in high-frequency systems, such as 5G and satellite communications, where the stability of oscillators is challenged by the increased susceptibility to thermal and flicker noise [7], [8], [9]. As carrier frequencies ascend into the millimeter-wave and Terahertz (THz) bands, even minor phase instabilities can lead to substantial performance losses, emphasizing the need for robust phase noise mitigation strategies. Recent research has increasingly focused on both understanding the fundamental mechanisms of phase noise and developing innovative techniques to mitigate its effects. Concurrently, digital compensation strategies have grown more sophisticated. In particular, adaptive algorithms now leverage real-time processing capabilities to dynamically mitigate phase noise effects during signal transmission and reception [10]. Interestingly, phase noise also brings more than negative effects to wireless systems. The inherent randomness and system-specific nature of phase noise make it a promising tool for enhancing system performance and security. In particular, phase noise can be used as a physical layer feature to authenticate received signals [11], [12] or identify transmitters [13], as a source for random number generation [14], and can aid in eavesdropping attack resistance [15].

Despite substantial advancements, the challenge of fully harnessing phase noise for useful applications while mitigating its negative impacts remains significant. Integrating phase noise characteristics into system design not only challenges conventional engineering paradigms but also offers a fertile ground for innovation in signal processing. There is a surprising lack of a comprehensive overview of the state-of-the-art estimation methods and applications of phase noise in wireless communications. This paper aims to bridge that gap by presenting an extensive survey on the models, estimation schemes, and applications of phase noise in wireless communications.

B. Comparative Analysis of Existing Surveys

Phase noise is a critical factor impacting the reliability of wireless communication systems. We provide an in-depth

review and comparison of existing surveys, highlighting their focus areas, classification perspectives, and limitations. This detailed analysis underscores the contributions and uniqueness of this paper.

Pankratz and Sánchez-Sinencio reviewed phase-noise models of integrated circuit oscillators and the negative effect of phase noise on system performance [16]. Different phase noises were modeled according to the noise modulation function, noise transfer function, and current-controlled oscillator gain, respectively. While the survey provides a deep theoretical analysis, it lacks discussions on practical implementations and contemporary applications. Furthermore, the techniques and models discussed are based on older technologies, which may not be applicable to recent systems. Leeson offered a historical overview of phase-noise studies, highlighting developments in oscillator phase-noise models over fifty years [17]. The focus is primarily on theoretical evolutions rather than applications to modern technologies. There is a lack of discussion on how these theoretical developments can be applied to contemporary wireless communication systems, and the paper does not cover advancements in phase noise mitigation techniques in modern systems. Hu et al. reviewed flicker phase-noise models and provided a tutorial on flicker noise in both nMOS-only oscillators and complementary oscillators [18]. However, the survey only considers specific types of phase noise and lacks discussion on how flicker noise impacts the performance of modern communication systems and the mitigation strategies that can be employed. Mohammadian and Tellambura reviewed models and estimation techniques for phase noise in wireless transceivers [9]. While the survey covers theoretical aspects well, it falls short in effectively linking these theoretical insights to practical applications within modern communication systems, particularly in demonstrating how these methodologies can be adapted to the evolving demands of newer wireless technologies. The survey does not provide a thorough analysis of the performance implications of phase noise in different system architectures. Mir and Buttar reviewed mitigation strategies within OFDM systems, emphasizing technical solutions to enhance performance [19]. Although comprehensive for OFDM, it lacks exploration of phase-noise effects and mitigation in other wireless systems, limiting its applicability to a broader array of technologies. The survey also does not provide a thorough comparison between different phase noise mitigation techniques. Easwaran and Krishnaveni reviewed the specific challenges of phase noise in millimeter wave systems, particularly for 5G applications [7]. The survey thoroughly addresses high-frequency system challenges but does not cover phase noise mitigation strategies or impacts in lower frequency ranges or older communication systems.

To facilitate understanding, we systematically compare the key surveys discussed above in Table I, including their primary focus, classification perspectives, and identified limitations. Building on the focused themes of existing surveys, our work offers a more comprehensive examination of phase noise across a broader spectrum of wireless communications. Unlike previous works that often concentrate on specific aspects or systems, our survey extensively covers the modeling, estimation, and mitigation of phase noise, while also exploring several unique applications, such as utilizing phase noise as a physical-layer feature for authenticating received signals. This broader scope not only enhances the understanding of phase noise as a research area but also introduces innovative perspectives on leveraging its inherent characteristics to improve system security and reliability. Our work aims to provide a holistic overview that can guide both current practices and future research in wireless communication technologies.

C. Contribution

We summarize the key contributions of this paper as follows.

1) We provide a comprehensive taxonomy framework for phase noise in terms of phase-noise models, phase-noise estimation and data reception schemes, joint transmit and receive designs with phase-noise mitigation, and applications of phase noise in wireless communications.

   a) We divide existing phase-noise models into four categories: phase-noise models associated with a Free-Running (FR) oscillator, phase-noise models associated with a Phase-locked Loop (PLL) circuit, phase-noise models associated with a channel estimator, and corresponding performance analysis of various phase-noise models. The phase-noise models associated with both an FR oscillator and a PLL circuit are further divided into white phase-noise models and colored phase-noise models based on the Power Spectral Density (PSD) of phase noise.

   b) We divide various phase-noise estimation and data reception schemes into three categories, based on how the phase noise is estimated and compensated by using classical estimation, Bayesian estimation, and machine-learning based estimation algorithms. Here, classical estimation algorithms include Maximum Likelihood (ML) and Least-Square (LS) algorithms, while Bayesian estimation algorithms encompass Minimum Mean Square Error (MMSE), Linear Minimum Mean Square Error (LMMSE), and Maximum a Posterior (MAP) estimation algorithms.

   c) We divide various joint transmit and receive designs with phase-noise mitigation into two categories, based on whether a constraint is imposed on the adaptively designed modulation constellation or not.

   d) We enumerate four interesting applications of phase noise in wireless communications, including phase-noise based Physical Layer Authentication (PLA), phase-noise for transmitter identification, phase-noise based random number generation, and artificial phase-noise against eavesdropping.

2) We meticulously analyze the contributions and identify the limitations and issues of each work reviewed on phase noise. We also provide enlightening lessons and summaries that encapsulate key findings and observations from the research, offering valuable guidance for both current and prospective studies.

TABLE I
COMPARISONS OF EXISTING SURVEY PAPERS ABOUT PHASE NOISE

Authors	Year	Focus	Classification Perspectives	Limitations
Pankratz and Sánchez-Sinencio [16]	2014	Integrated circuit oscillators' phase noise	Technical and Component-specific	• Lacks practical application discussion • Outdated techniques
Mir and Buttar [19]	2015	Mitigation techniques in OFDM systems	Application-specific and Methodological	• Lacks exploration of phase-noise effects and mitigation in other wireless systems • Limited comparison between different phase noise mitigation techniques
Leeson [17]	2016	Historical overview of phase noise	Chronological and Developmental	• Focuses mainly on theoretical models without connecting to practical applications • Lack of challenges and solutions related to current and future research directions
Hu et al. [18]	2021	Flicker phase noise in oscillators	Component-specific and Theoretical	• Only considers specific types of phase noise • Lack of discussion on mitigation strategies available for modern communication systems
Mohammadian and Tellambura [9]	2021	Phase noise models in transceivers	System-based and Methodological	• Limited connection to practical applications • Lacks thorough analysis of the performance implications of phase noise in different system architectures
Easwaran and Krishnaveni [7]	2022	Phase noise in millimeter wave systems for 5G and beyond	Technology-specific and Future-focused	• Does not cover low-frequency communication systems • Lack of discussion on mitigation strategies, especially in diverse and heterogeneous network environments

3) We provide an expanded set of research directions and accompanying challenges and potential solutions related to phase noise.

D. Organization

The remainder of this paper is organized as follows. In Section II, we introduce the system model of wireless communication systems with phase noise. Section III discusses phase-noise models and performance analyses. Section IV covers data reception schemes with phase-noise estimation and mitigation. Section V considers existing joint transmit and receive designs with phase-noise mitigation. In Section VI, we discuss diverse applications of phase noise in wireless communications. Future research directions and concluding remarks are provided in Section VII. For facilitating reading, we provide a summary of all abbreviations in Table II.

II. SYSTEM MODEL

Let us consider a typical wireless system with phase noise, as illustrated in Fig. 1. There are three sources to introduce phase noises during an entire process of wireless communications, as illustrated in the gray blocks in Fig. 1, i.e., an imperfect FR oscillator at the transmitter, an imperfect FR oscillator or PLL circuit at the receiver, and a defective equalizer. At the transmitter, a source data $\mathbf{s}_d$ is processed subsequently through a source encoder, a channel encoder, and a Base-Band (BB) modulator to obtain $\mathbf{s}_e$, $\mathbf{s}_c$, and $\mathbf{s}_m$, respectively. Then, pilot symbols are periodically inserted into $\mathbf{s}_m$ to obtain $\mathbf{s}_i$, where the pilot is used to facilitate channel estimation and equalization at the receiver. Then, $\mathbf{s}_i$ is processed sequentially through a pulse matching filter and a Radio-Frequency (RF) modulator to obtain $s_i(t)$ and $s_t(t)$, respectively. Specifically, the pulse matching filter is used to suppress an Inter-Symbol Interference (ISI) and the RF modulator is used to modulate a BB signal into an RF carrier signal with the help of a local oscillator, where an imperfect FR oscillator introduces phase noise. At last, $s_t(t)$ is broadcasted over a wireless channel with transmission power P_s.

Through the wireless channel, the received signal at the receiver can be expressed as

$$s_r(t) = h(t)\sqrt{P_s}s_t(t)\cos(2\pi f_{\text{LO}}t + \phi_t(t)) + \omega(t), \quad (1)$$

where $h(t) = \eta(t)\exp(j\theta(t))$ is the channel response with the fading amplitude $\eta(t)$ and fading phase $\theta(t)$, f_{LO} is the frequency of the local oscillator, $\phi_t(t)$ is the phase noise generated by the FR oscillator at the RF modulator, and $\omega(t)$ is an additive noise at the receiver. At the receiver, the received signal $s_r(t)$ is first processed through an RF demodulator to obtain $\hat{s}_i(t)$. Then, $\hat{s}_i(t)$ is filtered and sampled with a sampling period T_s through a pulse matching filter to obtain $\hat{\mathbf{s}}_i$. Specifically, the RF demodulator demodulates $s_r(t)$ with the help of an FR oscillator or a PLL circuit, where an imperfect FR oscillator or PLL circuit introduces a phase noise, and the pulse matching filter is used to suppress the ISI. Then, to compensate for the channel fading, an equalizer is used to obtain $\hat{\mathbf{s}}_m$, where the equalizer consists of two steps: channel estimation and channel equalization.

TABLE II
LIST OF ABBREVIATIONS

| Abbreviations	Full Name	Abbreviations	Full Name
AGC	Automatic Gain Control	AMI	Achievable Mutual Information
ASIC	Application-Specific Integrated Circuits	APSK	Amplitude Phase Shift Keying
BB	Base-Band	BCRLB	Bayesian CRLB
BER	Bit Error Rate	BLUE	Best Linear Unbiased Estimator
BP	Belief Propagation	BPSK	Binary Phase Shift Keying
CDF	Cumulative Distribution Function	CFBMC-OQAM	Circular Filter-Bank Multi-Carrier Offset Quadrature Amplitude Modulation
CFO	Carrier Frequency Offset	CNN	Convolutional Neural Network
CO-OFDM	Coherent Optical-OFDM	CPE	Common Phase Error
CRLB	Cramér-Rao Lower Bound	CW	Continuous Wave
CW-ML	Complex-Weighted ML	DNN	Deep Neural Network
ECM	Expectation Conditional Maximization	EKF	Extended Kalman Filter
EMPP	Enhanced Multiple Phase Noises	EP	Expectation Propagation
FIM	Fisher Information Matrix	FR	Free-Running
FSO	Free-Space Optical	GAMP	Generalized Approximate Message Passing
GFDM	Generalized Frequency Division Multiplexing	HCRLB	Hybrid CRLB
ICI	Inter Carrier Interference	ISF	Impulse Sensitivity Function
ISI	Inter-Symbol Interference	LFSR	Linear Feedback Shift Register
LMMSE	Linear Minimum Mean Square Error	LPF	Low Pass Filter
LS	Least-Square	LUT	Look-Up Table
MAP	Maximum a Posterior	MED	Minimum Euclidean Distance
MF	Mean Field	MIMO	Multiple-Input Multiple-Output
ML	Maximum Likelihood	MMSE	Minimum Mean Square Error
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor	MP	Message Passing
MPP	Multiple Phase Noises	MPSK	Multi Phase Shift Keying
MVU	Minimum Variance Unbiased	NOMA	Non-Orthogonal Multiple Access
OFDM	Orthogonal Frequency Division Multiplexing	OFDMA	Orthogonal Frequency-Division Multiple-Access
OFDM-IM	OFDM system combined with Index Modulation	PD	Probability of Detection
PFA	Probability of False Alarm	PLA	Physical-Layer Authentication
PLL	Phase-locked Loop	PSD	Power Spectral Density
QAM	Quadrature Amplitude	QPSK	Quadrature Phase Shift Keying
RF	Radio-Frequency	RO	Reference Oscillator
RVs	Random Variables	SA	Simulated Annealing
SC	Single Carrier	SDR	Software-Defined Radio
SDS	Smoother-Detector Structure	SEP	Symbol Error Probability
SER	Symbol Error Rate	SINR	Signal-to-Noise-plus-Interference Ratio
SIW	Self Interference Whitening	SNR	Signal-to-Noise Ratio
SP	Sum-Product	THz	Terahertz
TRNG	True Random Number Generator	VB	Variational Bayesian
VCO	Voltage Control Oscillator	VMP	Variational Message Passing
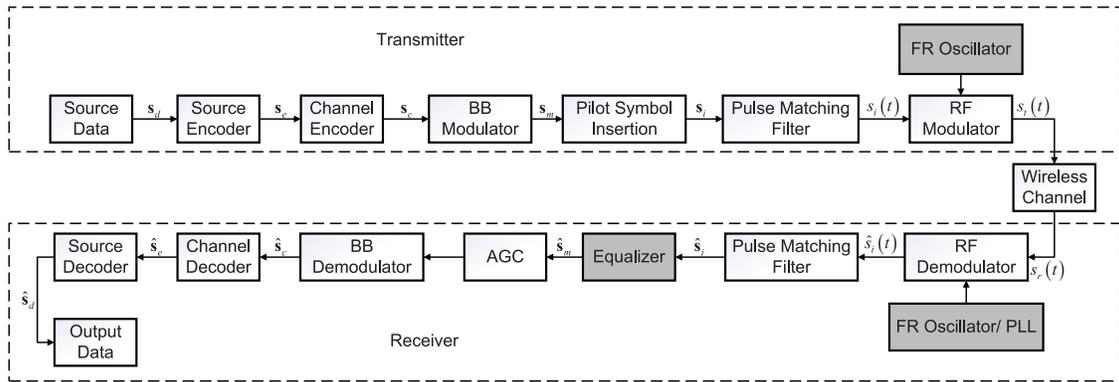
Fig. 1. Block diagram of a typical wireless system with phase noise.

- In the channel estimation, the receiver estimates the channel response as $\hat{h}(k) = \hat{\eta}(k)e^{j\hat{\theta}(k)}$ with the help of the pilot, where $\hat{h}(k)$ is the estimated version of $h(k)$ and $h(k)$ is the sampled output of $h(t)$ at the kth received symbol. Note that a flat block-fading channel is assumed such that channel response $h(k)$ is constant over the length of a frame. Estimation errors consist of two parts: amplitude error $\tilde{\eta}(k) = \hat{\eta}(k) - \eta(k)$ and phase error $\tilde{\theta}(k) = \hat{\theta}(k) - \theta(k)$. Note that, in this survey, the phase error $\tilde{\theta}(k)$ is treated as a phase noise introduced

by the equalizer. For analysis concision, the amplitude error $\tilde{\eta}(k)$ is ignored in this survey, since its effect can be compensated for via an Automatic Gain Control (AGC) circuit [20], [21].

- In channel equalization, based on $\hat{h}(k)$, the receiver equalizes the kth received symbol $\hat{s}_i(k)$ as

$$
\begin{aligned}
\hat{s}_m(k) &= \frac{\hat{h}^*(k)}{\left|\hat{h}(k)\right|^2}\hat{s}_i(k) \\
&= s_m(k)e^{j\left(\phi_t(k)+\phi_r(k)+\tilde{\theta}(k)\right)} + \tilde{\omega}(k), \quad (2)
\end{aligned}
$$

where $\phi_t(k)$ and $\phi_r(k)$ are the sampled outputs of $\phi_t(t)$ and $\phi_r(t)$ at the kth received symbol, respectively. Here, $\phi_r(t)$ is the phase noise introduced by the FR oscillator or PLL circuit at the RF demodulator and $\tilde{\omega}(k) = \frac{\hat{h}^*(k)}{|\hat{h}(k)|^2}\omega(k)$ is the residual receiver noise. From (2), we observe that an imperfect channel phase estimation also introduces a phase noise.

Then, an AGC is used to compensate for the amplitude error in channel estimation. At last, the output of the AGC is sequentially processed by a BB demodulator, a channel decoder, and a source decoder to obtain $\hat{\mathbf{s}}_c$, $\hat{\mathbf{s}}_e$, and $\hat{\mathbf{s}}_d$, respectively.

III. PHASE-NOISE MODELS

A. Overview

From Fig. 1, we observe that there are three modules that may introduce phase noises in a typical wireless system, which seriously affects the reception quality at the receiver. First, in the RF modulator module at the transmitter, the phase noise introduced by an FR oscillator is superimposed on the phase of the transmitted signal [1], [2], [3]. Second, in the RF demodulator module at the receiver, there are two ways to demodulate the received signal: an FR oscillator or a PLL circuit.

1) FR oscillator: since an FR oscillator at the receiver and an FR oscillator at the transmitter are independent, $\phi_r(t)$ and $\phi_t(t)$ are two independent Random Variables (RVs).
2) PLL circuit: since a PLL circuit tracks the carrier phase in real-time via a negative feedback loop, $\phi_r(t)$ is strongly correlated to $\phi_t(t)$, which makes the variance of $\phi_t(t) + \phi_r(t)$ using a PLL circuit much smaller than that using an FR oscillator.

Similar to the transmitter, the phase noise introduced by the FR oscillator or PLL circuit at the receiver is superimposed on the phase of the demodulated signal [3], [22], [23]. Third, in the equalizer module at the receiver, a phase noise is introduced by the channel estimation error, which is superimposed on the phase of the equalized symbols. In summary, the total phase noise is the sum of the phase noises of the above three modules, as presented in (2).

We summarize the taxonomy framework of existing phase-noise models in Fig. 2, which consists of phase-noise models associated with an FR oscillator, a PLL circuit, and a channel estimator, respectively, as well as the corresponding
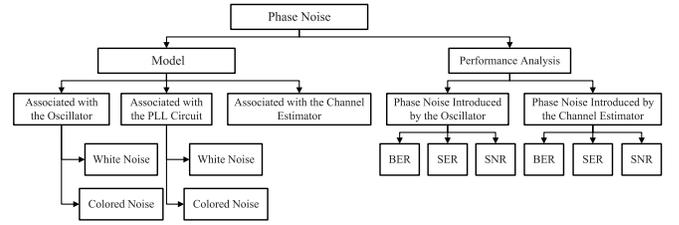

Fig. 2.   Taxonomy framework of existing phase-noise models.

performance analysis of various phase-noise models. The details of the taxonomy framework are given as follows.

B. Phase-Noise Models Associated With an Oscillator

1) Generic Phase-Noise Model: We denote $\phi_{\mathrm{FR}}(t)$ as the phase noise introduced by an FR oscillator. Referring to the IEEE Std 2414-2020 [24], phase noise $\phi_{\mathrm{FR}}(t)$ is quantified by the single-sideband Power Spectral Density (PSD), expressed as

$$
\mathcal{L}(f) = S_{\phi_{\mathrm{FR}}}(f)/2, \quad (3)
$$

where f is an offset frequency[1] from f_{LO} and $S_{\phi_{\mathrm{FR}}}(f)$ is the one-sided PSD of $\phi_{\mathrm{FR}}(t)$. Note that the value of $\mathcal{L}(f)$ becomes easy to be measured in practice when the following conditions are satisfied [24]. First, the variation of the signal amplitude is negligible. Second, the magnitude of $\phi_{\mathrm{FR}}(t)$ is sufficiently small, i.e., its root mean square value must be within 0.01 rad and its peak value must not exceed 0.2 rad. Based on measurements in [26], $S_{\phi_{\mathrm{FR}}}(f)$ can be expressed as

$$
S_{\phi_{\mathrm{FR}}}(f) = \sum b_{\beta_N} f^{\beta_N}, \quad (4)
$$

where b_{β_N} is the corresponding coefficient and β_N ranges from $-4 \leq \beta_N \leq 0$.

A generic phase-noise model is dependent on not only environmental parameters, e.g., temperature, humidity, pressure, magnetic field, and radiation, but also oscillator components, e.g., resistance and diode. A generic phase-noise model can be described as different noise models when offset frequencies are different. As the offset frequency increases, phase noise can be gradually modeled as a random walk frequency noise, flicker frequency noise, white frequency noise, flicker noise, and white noise, as illustrated in Fig. 3. A random walk frequency noise is usually related to environmental parameters, a flicker frequency noise may typically be related to environmental parameters and oscillator components, while all of the white frequency noise, flicker noise, and white noise are related to oscillator components. Note that random walk frequency noise, flicker frequency noise, and white frequency noise appear in some specific oscillators, whereas both white noise and flicker noise can be found in almost all oscillators. Therefore, this paper only focuses on both white noise and flicker noise.

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[1]In literature, there are two popular names for f, i.e., offset frequency [24], [25] or Fourier frequency [17], [25], [26].
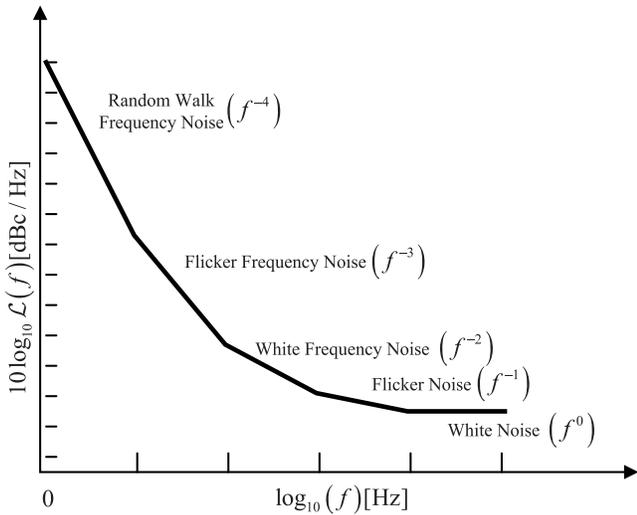
Fig. 3. PSD of a generic phase-noise model [26].

Fig. 4. PSDs of white noise and flicker noise models [26].

Fig. 5. High-level overview of phase-noise models associated with an oscillator.

Based on the results of [27], a white noise illustrated in Fig. 3 can be further divided into a thermal noise and a shot noise. The PSD of thermal noise is denoted as [27]

$$S_{\text{Thermal}}(f) = 4KTR, \tag{5}$$

where K is the Boltzmann's constant, T is the temperature (in Kelvin), and R is the resistance. The PSD of shot noise is denoted as [27]

$$S_{\text{Shot}}(f) = 2qI_{\text{D}}, \tag{6}$$

where q is the electron charge and I_{D} is the current through a junction. Based on the results of [28], the PSD of flicker noise is denoted as

$$S_{\text{Flicker}}(f) = \frac{k_f}{f}, \tag{7}$$

where k_f is a flicker noise coefficient of a particular device, e.g., $k_f = 10^{-12} \text{rad}^2/\text{Hz}$ in an electrical amplifier [28].

From (5) to (7), we observe that the PSDs of both a thermal noise and a shot noise are independent of the offset frequency, whereas the PSD of a flicker noise varies with the offset frequency, as illustrated in Fig. 4. Moreover, from Fig. 4, we can observe the following phenomena. First, the effect of a flicker noise dominates the phase-noise model at low offset frequency. Second, as the offset frequency increases, the effect of a flicker noise on the phase-noise model gradually declines, whereas the effect of both thermal noise and shot noise gradually dominates the phase-noise model. Third, the PSD of thermal noise is larger than that of a shot noise, which indicates a thermal noise has a greater impact on the phase-noise model as compared with a shot noise. Fourth, both thermal noise and shot noise can be described as a white-noise RV that has constant spectral density, e.g., [8], [28], [29], while a flicker noise should be described as a colored-noise RV that has variational spectral density, e.g., [8], [30], [31], [32], [33].

We present a high-level overview of phase-noise models associated with an oscillator in Fig. 5 to illustrate the details of specific components in an oscillator.
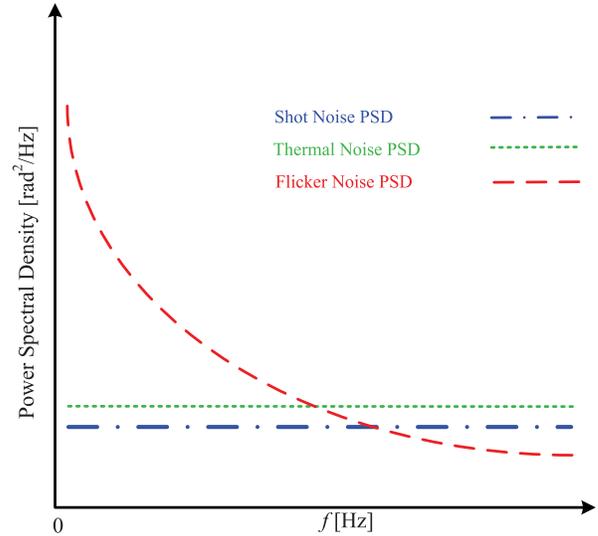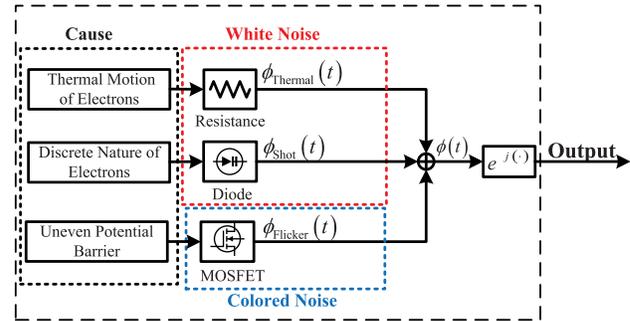
- Thermal noise is generated by passive components in an oscillator, e.g., resistances, due to the thermal motion of electrons or the Brownian motion of electrons [27].
- Shot noise is generated by active components in an oscillator, e.g., diodes, due to the discrete nature of electrons transfer across a potential barrier, e.g., a PN junction [27], [34], [35], [36].
- Flicker noise is generated by both passive and active components in an oscillator, e.g., Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET), due to the random capture and release of electrons in surface states and other traps [27].

2) White Phase-Noise Models: Leeson proposed a linear model to characterize the phase-noise spectrum [37], where the linear model is used to approximate the phase noise as its first-order Taylor expansion. However, the linear model [37] is less accurate in practical situations due to a large amount of deviation. In [29], Demir et al. analyzed the defects of the linear model [37] and proposed a nonlinear model, which applies to any oscillator described by differential equations. Specifically, a time shift $\alpha_{\text{FR}}(t)$ is used to replace $\phi_{\text{FR}}(t)$ for describing the effect of a phase noise, where their relationship

is expressed as

$$\phi_{\mathrm{FR}}(t) = 2\pi f_{\mathrm{LO}}\alpha_{\mathrm{FR}}(t). \tag{8}$$

Here, $\alpha_{\mathrm{FR}}(t)$ is modeled as a Wiener process, since FR oscillators can be treated as autonomous systems, which indicates that any perturbation in phase remains indefinitely. Based on [29], the value of $\alpha_{\mathrm{FR}}(t)$ is generally described as an asymptotically Wiener or Brownian motion process, expressed as

$$\alpha_{\mathrm{FR}}(t) = \sqrt{c}B(t), \tag{9}$$

where c is a parameter to assess the quality of the oscillator and $B(t)$ is a standard Wiener process. Since $B(t_2) - B(t_1) \sim \mathcal{N}(0,1)$, where t_1 and t_2 are two adjacent moments, respectively. The variance of a Wiener process increases linearly with time, expressed as

$$\sigma^2_{\alpha_{\mathrm{FR}}(t)} = ct. \tag{10}$$

Based on [3], the value of c can be expressed as

$$c = \frac{\beta}{2\pi f_{\mathrm{LO}}{}^2}, \tag{11}$$

where β is the one-sided 3-dB bandwidth of the Lorentzian spectrum of an oscillator and its value can be obtained directly by measurement. Then, in [38], Mehrpouyan et al. modeled $\phi_{\mathrm{FR}}(t)$ as a Wiener process, expressed as

$$\phi_{\mathrm{FR}}(t_2) = \phi_{\mathrm{FR}}(t_1) + \Delta_{\phi_{\mathrm{FR}}(t)}, \tag{12}$$

where t_2 is a moment after t_1. Note that, in (12), the value of t_2 may be much higher than that of t_1 rather than two adjacent moments. Here, $\Delta_{\phi_{\mathrm{FR}}(t)}$ denotes the increment of $\phi_{\mathrm{FR}}(t)$ and its variance can be expressed as

$$\sigma^2_{\Delta_{\phi_{\mathrm{FR}}(t)}} = 2\pi\beta|t_2 - t_1|. \tag{13}$$

According to (11), we have

$$\sigma^2_{\Delta_{\phi_{\mathrm{FR}}(t)}} = 2\pi\beta t = 4\pi^2 f_{\mathrm{LO}}{}^2 ct = 4\pi^2 f_{\mathrm{LO}}{}^2 \sigma^2_{\alpha_{\mathrm{FR}}(t)}. \tag{14}$$

Ham and Hajimiri introduced the concept of virtual damping to explain the oscillator spectral spread due to phase noise [39], which is useful to describe both the oscillator and resonator in a unified manner. Specifically, an alternative expression for $\sigma^2_{\Delta_{\phi_{\mathrm{FR}}(t)}}$ in (14) is presented, expressed as

$$\sigma^2_{\Delta_{\phi_{\mathrm{FR}}(t)}} = 2D_{\mathrm{B}}t, \tag{15}$$

where D_{B} is the virtual damping rate, indicating how fast the phase diffusion occurs. Moreover, in [39], Ham and Hajimiri took an LC oscillator as an example to analyze the relationship between the phase noise and the Einstein relation, which provides insight into the relationship between the phase noise variance and the parameters of an oscillator. Specifically, the virtual damping rate can be regarded as the diffusion coefficient of the Brownian particle according to the Einstein relation [40], expressed as

$$D_{\mathrm{B}} = \frac{KT}{m_{\mathrm{B}}} \cdot \frac{1}{\gamma_{\mathrm{B}}}, \tag{16}$$

where m_{B} is the mass of the particle and $\frac{1}{\gamma_{\mathrm{B}}}$ is the friction factor. Based on the parameters of the LC oscillator, (16) is further derived as

$$D_{\mathrm{B}} = \frac{1}{V_0^2} \cdot \frac{KT}{C} \cdot \frac{1}{RC}, \tag{17}$$

where V_0 is the supply voltage and C is the capacity.

3) Colored Phase-Noise Models: Demir analyzed the stochastic characterization of the phase noise in oscillators and described it as a general colored noise [41]. Tchamov et al. presented a linear time-invariant model of a charge-pump PLL by fully considering the contributions of both white noise and colored noise due to phase noise [42], which refines the results of [41] to a specific flicker noise. Specifically, in [42], Tchamov et al. used $\mathcal{V}(f)$ to describe the phase noise as

$$\mathcal{V}(f) = 10\log_{10}\left(\frac{f_{\mathrm{LO}}{}^2(c_{\mathrm{T}} + c_{\mathrm{F}}S_{\mathrm{Flicker}}(f))}{\pi^2 f_{\mathrm{LO}}{}^4(c_{\mathrm{T}} + c_{\mathrm{F}}S_{\mathrm{Flicker}}(f))^2 + f^2}\right), \tag{18}$$

where both c_{T} and c_{F} are constants to characterize the contributions of white noise (i.e., thermal noise) and colored noise (i.e., flicker noise), respectively. Since, from (7), we observe that $S_{\mathrm{Flicker}}(f)$ cannot converge at $f = 0$, a cutoff frequency γ_c is introduced to attain a finite value at $f = 0$ [35]. Then, $S_{\mathrm{Flicker}}(f)$ is transformed as [41]

$$\begin{aligned} S_{\mathrm{Flicker}}(f) &= 4\int_{\gamma_c}^{\infty}\frac{1}{\gamma^2 + (2\pi f)^2}d\gamma \\ &= \frac{1}{|f|} - 4\frac{\arctan\left(\frac{\gamma_c}{2\pi f}\right)}{2\pi f}. \end{aligned} \tag{19}$$

Thus, $S_{\mathrm{Flicker}}(f)$ at $f = 0$ can be denoted as

$$S_{\mathrm{Flicker}}(0) = \frac{4}{\gamma_c}. \tag{20}$$

Jin et al. presented an analysis of an LC oscillator [8], which quantifies the effect of both flicker noise and thermal noise based on the Impulse Sensitivity Function (ISF). Qi et al. presented an analysis of an optoelectronic oscillator by considering white noise, flicker noise, and laser frequency noise [28], where a directly modulated distributed feedback laser is established. For special oscillators, if flicker-noise up-conversion mechanisms are used, a flicker noise can be transformed into other forms after passing through an oscillator loop [43]. Pepe et al. analyzed a flicker-noise up-conversion mechanism for voltage-biased oscillators [32], where a flicker noise is upconverted to a flicker frequency noise, and the effect of an up-conversion mechanism on a flicker noise is named the Groszkowski effect. Moreover, in [32], an amplitude modulation to pulse modulation conversion noise is discussed quantitatively. Based on the results of [32], Pepe et al. designed a 65-nm CMOS Voltage Control Oscillator (VCO) with suppressed up-conversion flicker noise [33], where the loop delay caused by stray capacitances at the drain node of the transistors is discussed quantitatively. Since the analyses in [32], [33] are provided for a particular oscillator, Pepe and Andreani further presented an analysis of a generic harmonic oscillator [30], which analyzes a flicker-noise up-conversion
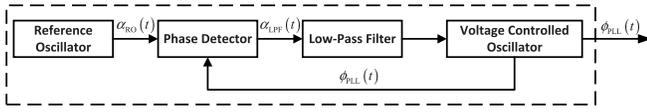
Fig. 6. Phase noise introduced by a simple PLL circuit.

mechanism in a harmonic oscillator, where a flicker noise is upconverted to a white frequency noise.

4) Lessons: We have reviewed some representative phase-noise models associated with an oscillator. From this review, we gather the following lessons:

- The phase noise introduced by an oscillator is the result of hardware defects, which cannot be avoided completely but can be reduced by improving the device quality or switching to a better oscillator design [32], [33].
- Since the phase noise introduced by an oscillator accumulates over time, its variance becomes larger over time, as shown in (14).
- Phase noise can be described as white noise and flicker noise in almost all oscillators [29], [39], [41], but, in some special oscillators, the phase noise can be described in other forms. For instance, in an optoelectronic oscillator, the phase noise can be described as a sum of white noise, flicker noise, and laser frequency noise [28], where the laser frequency noise is introduced by small-signal direct modulation.

C. Phase-Noise Models Associated With a PLL Circuit

1) White Phase-Noise Models: We denote $\phi_{\mathrm{PLL}}(t)$ as the phase noise introduced by a simple PLL circuit, as illustrated in Fig. 6. Specifically, a phase detector compares the phases of the output signals of a Reference Oscillator (RO) and a VCO to generate a phase-error signal. The phase-error signal is processed through a Low Pass Filter (LPF) to control the frequency of VCO to be consistent with that of the RO. Since both the RO and the VCO in the PLL circuit can be regarded as FR oscillators, the above analyses of the FR oscillator can be applied as well.

Mehrotra introduced stochastic differential equations to describe the phase noise of a PLL circuit [44], where a time shift $\alpha_{\mathrm{PLL}}(t)$ is used to replace $\phi_{\mathrm{PLL}}(t)$ for describing the effect of a phase noise. The relationship between $\alpha_{\mathrm{PLL}}(t)$ and $\phi_{\mathrm{PLL}}(t)$ is expressed as

$$\phi_{\mathrm{PLL}}(t) = 2\pi f_{\mathrm{VCO}}\alpha_{\mathrm{PLL}}(t), \tag{21}$$

where f_{VCO} is the frequency of the VCO. Specifically, the value of $\alpha_{\mathrm{PLL}}(t)$ is denoted as [44]

$$\alpha_{\mathrm{LPF}}(t) = \alpha_{\mathrm{PLL}}(t) - \alpha_{\mathrm{RO}}(t), \tag{22}$$

where $\alpha_{\mathrm{RO}}(t)$ is the time shift of the RO, modeled as a Wiener process, and $\alpha_{\mathrm{PLL}}(t)$ is the time shift of the input of the LPF, modeled as an appropriate multidimensional Ornstein-Uhlenbeck process. In the perfect state, f_{VCO} is locked to f_{RO}, i.e., $f_{\mathrm{VCO}} = f_{\mathrm{RO}}$, where f_{RO} is the frequency of the
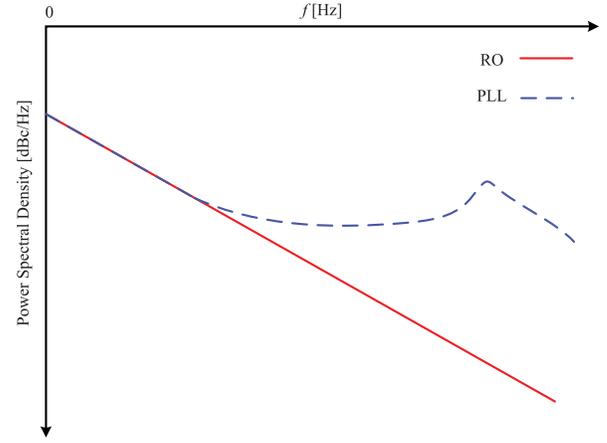

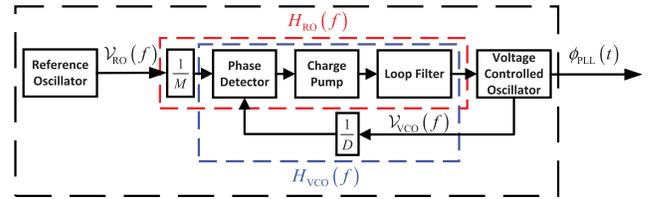Fig. 7. PSD of the phase noises introduced by a simple PLL circuit [44].

Fig. 8. Overview of the phase noise introduced by a 3rd-order charge-pump PLL circuit.

RO. The correlation between $\alpha_{\mathrm{RO}}(t)$ and $\alpha_{\mathrm{PLL}}(t)$, and the autocorrelation of $\alpha_{\mathrm{PLL}}(t)$ are respectively denoted as

$$E\{\alpha_{\mathrm{LPF}}(t_1)\alpha_{\mathrm{RO}}(t_2)\} = \sum_{i=1}^{n_0} \mu_i e^{\lambda_i \min(0, t_2 - t_1)}, \tag{23}$$

and

$$E\{\alpha_{\mathrm{LPF}}(t_1)\alpha_{\mathrm{LPF}}(t_2)\} = \sum_{i=1}^{n_0} v_i e^{-\lambda_i |t_2 - t_1|}, \tag{24}$$

where $n_0 = 1 + o_{\mathrm{LPF}}$ being the incremented order of o_{LPF}. The calculation processes of parameters μ_i, λ_i, and v_i in (23) are described in [44]. The PSD of the phase noise introduced by a simple PLL circuit is illustrated in Fig. 7.

2) Colored Phase-Noise Model: Tchamov et al. derived the PSD of $\phi_{\mathrm{PLL}}(t)$ in a 3rd-order charge-pump PLL circuit, denoted as $S_{\phi_{\mathrm{PLL}}}(f)$ [42], as illustrated in Fig. 8. Since Tchamov et al. only considered the effect of the VCO and RO but ignored that of the remaining electronics. $S_{\phi_{\mathrm{PLL}}}(f)$ is modeled as a linear model, expressed as

$$S_{\phi_{\mathrm{PLL}}}(f) = |H_{\mathrm{VCO}}(f)|^2 \mathcal{V}_{\mathrm{VCO}}(f) + |H_{\mathrm{RO}}(f)|^2 \mathcal{V}_{\mathrm{RO}}(f), \tag{25}$$

where $\mathcal{V}_{\mathrm{VCO}}(f)$ is the PSD of the phase noise in the VCO and $\mathcal{V}_{\mathrm{RO}}(f)$ is the PSD of the phase noise in the RO. Note that both $\mathcal{V}_{\mathrm{VCO}}(f)$ and $\mathcal{V}_{\mathrm{RO}}(f)$ have been defined in (18) with different parameters. Here, $H_{\mathrm{VCO}}(f)$ and $H_{\mathrm{RO}}(f)$ are the transfer functions of the VCO and RO, respectively, expressed as

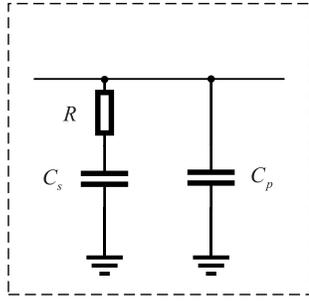$$H_{\mathrm{VCO}}(f) = \frac{j2\pi fD}{j2\pi fD + I_p K_{\mathrm{VCO}} H_{\mathrm{LF}}(f)}, \tag{26}$$
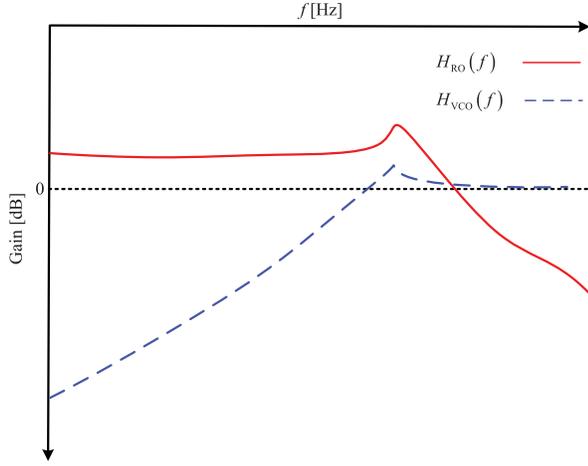
Fig. 9. A diagram of the loop filter [42].

Fig. 10. Loop transfer functions of the RO and the VCO [42].

and

$$H_{\text{RO}}(f) = \frac{DI_p K_{\text{VCO}} H_{\text{LF}}(f)}{M\left(j2\pi fD + I_p K_{\text{VCO}} H_{\text{LF}}(f)\right)}, \qquad (27)$$

where $\frac{1}{D}$ and $\frac{1}{M}$ denote the integer frequency dividers of VCO and RO, respectively, I_p is the current of the charge pump, and $\frac{1}{K_{\text{VCO}}}$ is the integration time constant of the VCO. Here, $H_{\text{LF}}(f)$ is the transfer function of a loop filter, expressed as

$$H_{\text{LF}}(f) = \frac{1 + j2\pi fRC_s}{j2\pi f\left(C_s + C_p + j2\pi fRC_p C_s\right)}, \qquad (28)$$

where R is the resistance of the loop filter. Here, C_s and C_p are the capacitors of the loop filter, respectively. A diagram of the loop filter is illustrated in Fig. 9. Note that the output of the VCO undergoes a high-pass filter, whereas the output of the RO undergoes a low-pass filter and an amplifier. Fig. 10 illustrates the loop transfer functions of the RO and the VCO, while Fig. 11 illustrates the PSDs of the outputs of the RO, the VCO, and the PLL affected by the phase noise.

3) Lessons: We summarize the lessons on the phase-noise models introduced by a PLL circuit as follows:

- The phase noise introduced by a PLL circuit is the result of hardware defects as well, which cannot be avoided completely but can be reduced by improving the PLL circuit/device quality or changing the PLL design [42].
- The phase noises generated by different components of a PLL circuit are uncorrelated and the total phase noise
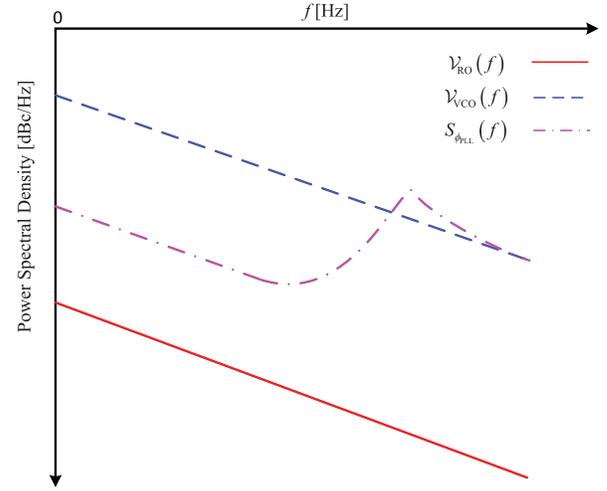

Fig. 11. PSDs of the phase noises introduced by a 3rd-order charge-pump PLL circuit [42].

is their sum at the output after passing through a loop filter [42].

D. Phase-Noise Models Associated With Channel-Estimation Errors

1) Overview: In literature, e.g., [45], [46], [47], [48], [49], [50], [51], channel-estimation errors are often modeled as complex zero-mean Gaussian RVs [45], [46], [47], [48], [49], [50], [51]. Tang et al. derived the Probability Density Function (PDF) of $\tilde{\theta}(t)$ in Rayleigh fading channels [46]. Since both the channel response and channel estimation error are modeled as complex zero-mean Gaussian RVs, the joint distribution of $\theta(t)$ and its estimate $\hat{\theta}(t)$ can be modeled as [46]

$$f\left(\theta, \hat{\theta}\right) = \frac{1 - \rho_\eta}{4\pi^2}\left[\frac{\left(1 - q_\eta\right)^{\frac{1}{2}} + q_\eta\left(\pi - \cos^{-1}\left(q_\eta\right)\right)}{\left(1 - q_\eta^2\right)^{\frac{3}{2}}}\right], \qquad (29)$$

where $q_\eta = \sqrt{\rho_\eta}\cos(\theta - \hat{\theta})$ and $\rho_\eta = \frac{\text{cov}(\eta^2, \hat{\eta}^2)}{\sqrt{\text{var}(\eta^2)\text{var}(\hat{\eta}^2)}}$. Based on (29), the PDF of $\tilde{\theta}(t)$ is derived as

$$f\left(\tilde{\theta}\right) = \frac{1 - \rho_\eta\left(2\pi - \left|\tilde{\theta}\right|\right)}{4\pi^2}\left[\frac{1}{1 - \rho_\eta\cos^2\left(\tilde{\theta}\right)}\right.$$
$$\left. + \frac{\sqrt{\rho_\eta}\cos\left(\tilde{\theta}\right)\left(\pi - \cos^{-1}\left(\sqrt{\rho_\eta}\cos\left(\tilde{\theta}\right)\right)\right)}{\left(1 - \rho_\eta\cos^2\left(\tilde{\theta}\right)\right)^{\frac{3}{2}}}\right]. \qquad (30)$$

2) Lessons: Useful lessons pertaining to the phase-noise model associated with channel-estimation errors are

- The phase noise introduced by channel-estimation errors results from both channel impairments, e.g., fading and noise, and errors of channel-estimation algorithms, which cannot be avoided completely but can be reduced by improving the channel-estimation algorithm.

- The distribution of the phase noise introduced by channel-estimation errors is determined by the channel-fading distribution.
- Unlike the phase noise introduced by an oscillator or a PLL circuit, the phase noise introduced by channel estimation errors does not accumulate over time.

E. Summary of Phase-Noise Models

We summarize the above phase-noise models and their limitations and issues in Table III. In comparison with the phase-noise models associated with an oscillator and a PLL circuit, there is a lack of research on the phase-noise models associated with a channel estimator. Some of the results of phase-noise models associated with an oscillator can be extended to the phase-noise models associated with a PLL circuit, but it is more difficult to model the rest of the colored phase noises other than flicker noise due to the more complex circuit structure of the PLL circuit and the tentatively undefined phase-noise characteristics.

While this survey focuses on wireless systems, several discussed noise models can be adapted for wired communication scenarios. Phase noise, primarily associated with oscillator and PLL circuit imperfections, affects signal integrity in both wireless and wired contexts. In wired systems such as fiber-optic and coaxial cable communications, the stability of oscillators and the performance of PLL circuits are critical for maintaining high-quality signal transmission over long distances. The phase noise models associated with these components fundamentally describe the fluctuations in the phase of a signal, which can lead to synchronization issues and increased bit error rates in both wireless and wired systems. The generic models for oscillator-induced phase noise and PLL circuit phase noise describe the behavior of these components under various operational conditions and noise influences, such as thermal and flicker noise. These models are applicable to wired communications since the fundamental principles governing oscillator behavior and PLL performance do not change significantly between wired and wireless systems. Moreover, wired systems often require even more precise control over phase noise due to typically higher data rates and longer transmission distances. Therefore, it is critical to employ suitable modeling approaches and mitigation strategies for enhancing the robustness of these systems against phase-related errors.

F. Performance Analyses in the Presence of Phase Noise

The effect of the phase noise on the reception performance can be evaluated by various metrics, e.g., SNR, BER, and Symbol Error Rate (SER). In literature, based on the above phase-noise models, different performance analyses for different communication systems have been provided.

1) Performance Analyses With Phase Noise Introduced by an Oscillator: Pollet et al. discussed the effects of the phase noise on both the SNR and BER in OFDM and Single Carrier (SC) systems [4], where perfect phase-noise compensation is assumed. Specifically, closed-form expressions for the SNR degradation due to the phase noise in OFDM and SC systems

are derived, where the phase noise is modeled as a Wiener process. Then, Armada relaxed the assumption of perfect phase-noise compensation in [4] and discussed the effects of the phase noise on both the SER and SNR in OFDM systems [5]. Specifically, a closed-form expression for the SNR degradation due to the phase noise is derived, where the phase-noise variance is assumed to be small, i.e., much less than 1, and known [5]. Pollet et al. generalized the results of [4] to three different receivers in OFDM systems: a coherent receiver, a Common Phase Error (CPE) correction receiver, and a differential receiver [6], where the CPE correction receiver further corrects the CPE of the received signal. Specifically, closed-form expressions of the SNR degradation due to the phase noise for the above three receivers are derived, where the SNR degradation is expressed as a function of the phase-noise spectrum. Moreover, in [6], the impacts of fast, moderate, and slow phase noise on the SER are investigated, where the classification criterion is defined as the ratio of the phase-noise bandwidth to the OFDM subcarrier spacing, i.e., the larger the ratio, the faster the phase noise varies. Since the analyses in [4], [5] are only valid in the scenario with a small phase noise, Wu and Bar-Ness further derived a closed-form expression of the Signal-to-Noise-plus-Interference Ratio (SINR) in OFDM systems with given parameters [52], i.e., the phase-noise bandwidth β, number of subcarriers, and transmission-data rate, where various possible phase noise levels and different subcarrier numbers are considered. Since Orthogonal Frequency-Division Multiple-Access (OFDMA) is based on OFDM, Steendam et al. generalized the results of [4] to an OFDMA system [53]. Specifically, closed-form expressions for the SNR degradation due to the phase noise in OFDMA, OFDM, and conventional FDMA systems are derived and compared, where the phase-noise spectrum is assumed to be known. Piemontese et al. examined the impact of phase noise on communication systems where local oscillators provide reference signals for carrier and timing synchronization [54]. They derived closed-form expressions for the SNR degradation and ISI in discrete-time phase noise channels. The study focuses on evaluating power loss and performance degradation due to oscillator non-idealities in both free-running and phase-locked oscillators, providing insights into practical system performance by analyzing the PSD of the oscillator phase noise. Wang et al. derived new mathematical models to analyze the impact of phase noise on sidelobe cancelation systems, particularly when using distributed PLLs [55]. They derived closed-form expressions for SINR degradation in sidelobe cancellation systems with distributed PLLs. This study emphasizes the necessity of considering phase noise in the performance evaluation of beamforming systems and verifies the models through signal-level simulations.

2) Performance Analyses With Phase Noise Introduced by Channel-Estimation Errors: Wilson and Cioffi derived closed-form expressions of conditional PDFs of the received signals in Rayleigh and Rician Channels with channel-estimation errors [56], where a deterministic transmit signal is considered. Moreover, based on the above PDFs, they analyzed the BER and the SER of both M-ary Phase Shift Keying (MPSK)

TABLE III
PHASE-NOISE MODELS AND RELATED LIMITATIONS AND ISSUES

Authors	Phase-Noise Models	Contributions	Open Issues
Demir et al. [29], and Ham and Hajimiri [39]	White phase-noise models associated with an oscillator	White phase-noise models 1) based on nonlinear analysis [29] 2) based on the concept of virtual damping [39]	Colored noise not considered
Demir [41], Tchamov et al. [42], Pepe et al. [32], Pepe et al. [33], Pepe and Andreani [30], Qi et al. [28], and Jin [8]	Colored phase-noise models associated with an oscillator	Colored phase-noise models 1) based on nonlinear analysis [41] 2) for an FR oscillator [42] 3) for a voltage-biased oscillator [32] 4) for a CMOS VCO [33] 5) for a trans-conductor-based harmonic oscillator [30] 6) using a directly modulated distributed feedback laser [28] 7) based on the ISF [8]	The random walk frequency noise cannot be modeled with these models.
Mehrotra [44]	White phase-noise models associated with a PLL circuit	A white phase-noise model based on nonlinear analysis	Colored noise not considered
Tchamov et al. [42]	Colored phase-noise models associated with a PLL circuit	A colored phase-noise model in a 3rd-order charge-pump PLL circuit	Only considered the flicker noise
Tang et al. [46]	Phase-noise models associated with a channel estimator	A phase-noise model for Multilevel Quadrature Amplitude Modulation (M-QAM)	Only considered the Rayleigh fading channel

and MQAM systems [56]. Wilson and Cioffi generalized the results of [56] to an arbitrary two-dimensional constellation [57], where closed-form expressions of the SER with channel-estimation errors in Rayleigh fading channels are derived. Moreover, they further investigated the impact of different channel-estimation algorithms on the reception of different constellations [57]. Since Rayleigh and Rician channels [56], [57] can be treated as special cases of Nagakami-m channels [58], Smadi et al. derived the closed-form expressions of the BER of Binary Phase Shift Keying (BPSK) and Quadrature Phase Shift Keying (QPSK) systems with channel-estimation errors in Nagakami-m fading channels [58].

For Free-Space Optical (FSO) communication systems, Han et al. analyzed the channel-estimation error in a Fisher-Snedecor $\mathcal{F}$ fading channel [59]. Specifically, closed-form expressions of both the PDF and Cumulative Distribution Function (CDF) of the SNR are derived, along with expressions of the outage probability, BER, and ergodic capacity. Moreover, the asymptotic expressions for the outage probability and BER are obtained to provide insights into the FSO system performance in a high SNR regime [59].

3) Lessons: The results and related limitations and issues pertaining to performance analysis in the presence of channel and phase estimation errors are summarized in Table IV. From this review, we gather the following lessons:

- OFDM systems are more sensitive to phase noise than SC systems, where the effect of the phase noise can be mitigated by choosing an appropriate signal transmission scheme according to different phase-noise levels [4].
- QPSK systems are more sensitive to phase noise than BPSK systems, where the effect of the phase noise can be reduced by choosing an appropriate modulation scheme according to different actual phase-noise levels [58].
- The SNR degradation in both OFDM and OFDMA systems depends on the variance of the phase noise

rather than the spectral shape, e.g., bandwidth and roll-off, whereas the SNR degradation in traditional FDMA systems depends on the spectral shape [53].
- Practical system performance can be significantly impacted by oscillator imperfection. Considering these effects in models can provide more accurate insights for evaluating performance degradation and developing effective mitigation strategies [54].
- Distributed PLLs in beamforming systems introduce additional phase noise that must be accounted for to accurately predict performance metrics such as SINR and beamforming gain [55].

G. Summary

There are three modules that can introduce phase noises in a typical wireless system, including the RF modulator module at a transmitter, the RF demodulator module, and the equalizer module at a receiver. The phase noises introduced by both the RF modulator and RF demodulator are mainly due to the FR oscillators and PLL circuits, and they can be modeled as white-noise RVs and colored-noise RVs [8], [28], [29]. The phase noises introduced by the equalizer are mainly contributed by a channel estimator, where the phase noises can be modeled as complex zero-mean Gaussian RVs [45], [46], [47], [48], [49], [50], [51]. The phase noise cannot be avoided completely, but its negative effect can be reduced to a certain extent according to the following ways:

- Improve the hardware quality of the oscillator and PLL circuit;
- Select an appropriate topology and design for the oscillator and PLL circuit;
- Improve the performance of the employed channel-estimation algorithm;
- Compensate for the phase deviation in the received signal caused by phase noise before demodulating it;

TABLE IV
PERFORMANCE ANALYSES FOR PHASE-NOISE MODELS

Authors	Year	Sources of Phase Noise	Channel	Performance Metrics	Contributions	Open Issues
Pollet *et al.* [4]	1995	Oscillator	AWGN	SNR BER	Expressions of both SNR degradation and approximate BER degradation of OFDM systems	The phase noise is assumed to be perfectly compensated for and channel fading is not considered.
Steendam *et al.* [53]	1998	Oscillator	AWGN	SNR	Expression of SNR degradation of OFDMA systems	Channel fading not considered
Wilson and Cioffi [56]	1999	Channel estimation error	Rayleigh and Rician Channels	BER SER	Expressions of both BER and SER of M-QAM in both Rayleigh and Rician channels	The results are only applicable to low-order QAM.
Armada *et al.* [5]	2001	Oscillator	AWGN	SNR SER	Expression of SNR degradation of OFDM systems along with corresponding SER simulation	Channel fading not considered
Piazzo *et al.* [6]	2002	Oscillator	AWGN	SNR SER	Expressions of SNR degradation for three different OFDM receivers	Channel fading not considered
Dong and Beaulieu [57]	2003	Channel estimation error	Rayleigh Channel	SER	Expression of SER of arbitrary constellation order in a Rayleigh channel	Only considered a Rayleigh channel
Wu *et al.* [52]	2004	Oscillator	Fading Channel	SINR	Expression of SINR of OFDM systems	Only considered SINR
Smadi *et al.* [58]	2012	Channel estimation error	Nakagami-m Channel	BER	Expressions of BER of both BPSK and QPSK systems in Nakagami-m channel	Only considered BER
Han *et al.* [59]	2022	Channel estimation error	$\mathcal{F}$ Turbulence Channel	SNR BER	Expressions of both SNR and BER of FSO communication systems in a Fisher-Snedecor $\mathcal{F}$ fading channel	Only considered on-off keying modulation
Wang *et al.* [55]	2023	Oscillator	AWGN	SINR	Expressions of SINR degradation in sidelobe cancellation systems with distributed PLLs	Focused only on distributed PLLs and satellite communication systems
Piemontese *et al.* [54]	2024	Oscillator	AWGN	SNR ISI	Expressions of SNR degradation and intersymbol interference in discrete-time phase noise channels	Limited to discrete-time models and specific oscillator parameters

- Improve the performance of the signal-transmission scheme.

IV. PHASE-NOISE ESTIMATION AND DATA RECEPTION SCHEMES

A. Overview

To mitigate the negative effect of the phase noise, various phase-noise estimation and data reception schemes were designed, as illustrated in Fig. 12, where their employed estimation algorithms are highlighted. In the literature, the estimation algorithms for the phase noise can be divided into three categories: classical, Bayesian, and machine-learning based estimation algorithms.

- In classical estimation algorithms, represented by the ML and LS algorithms, the estimation performance is mainly determined by the observed samples at the receiver.
- In Bayesian estimation algorithms, represented by the MMSE, LMMSE, and MAP algorithms, the estimation performance is determined by both the observed samples at the receiver and the prior knowledge of the phase noise. In comparison with the MMSE algorithm, the LMMSE algorithm is a linear estimation algorithm by relaxing some restriction of the prior knowledge, e.g., the PDF of the phase noise, where only the second-order moments of the phase noise are required. Since the prior knowledge of the phase noise is fully utilized, the Bayesian estimation algorithms have better performance than the classical estimation algorithms.
- In machine-learning based estimation algorithms, represented by the Deep Learning (DL) algorithms,

the estimation performance is mainly determined by the observed samples at the receiver as well. In comparison with the classical estimation algorithms, machine-learning based estimation algorithms eliminate the complex iterative process, while, compared to the Bayesian estimation algorithms, they do not require additional prior knowledge. However, machine-learning based estimation algorithms cannot provide an explicit relationship between the system model and the parameters of phase noise.

Note that the phase-noise estimation is often carried out with either channel estimation or Carrier Frequency Offset (CFO) estimation.

There are three kinds of prior knowledge used in various data reception schemes, i.e., prior knowledge of the data symbols, prior knowledge of the channel, and prior knowledge of the phase noise. We divide various data reception schemes into three types: direct reception, stepwise reception, and decision-feedback reception.

- In direct reception where phase-noise estimation is omitted, based on the prior knowledge of the phase noise, e.g., the PDF of the phase noise, a receiver directly performs data reception by employing ML or MAP algorithms. Note that, in this scenario, the MAP algorithm further requires the prior knowledge of the data symbols, whereas the ML algorithm does not.
- In stepwise reception, a receiver first performs channel and phase-noise estimation with the help of the received pilot. Then, based on the channel and phase-noise estimates, the receiver performs data reception. Note that, in some scenarios of stepwise reception, the CFO estimation
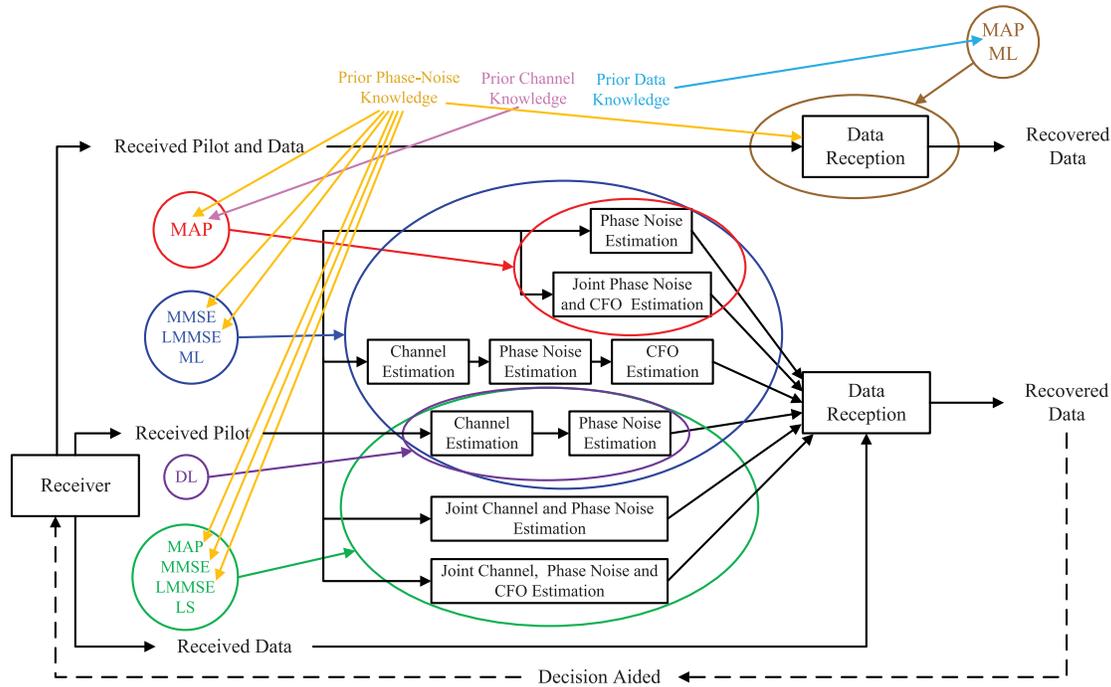
Fig. 12. Overview of various data reception schemes with phase-noise estimation and compensation, where their employed estimation algorithms are highlighted.

also needs to be considered prior to data reception. If the prior knowledge of the phase noise is available, phase-noise estimation employs the MAP, MMSE, or LMMSE algorithms. If such prior knowledge is not available, phase-noise estimation employs the ML, LS, or DL algorithms. In some simplified scenarios where channel estimation is omitted, based on the prior knowledge of the channel and phase noise, a receiver performs phase-noise estimation by employing the MAP algorithm.

- In decision-feedback reception, similar to the second category, the data reception schemes are designed based on a decision-feedback mechanism, as illustrated by the dotted lines in Fig. 12. Specifically, the output of the data reception is feedbacked to the input of the phase-nose estimation to improve the estimation performance.

Note that, the first type is suitable for the scenario where the data reception performance is only considered, while the result of a phase-noise estimation is not important. Moreover, the second type is suitable for the scenario that the phase noise varies slowly, while the third type is suitable for the opposite scenario.

B. Classical Estimation Algorithms

Classical estimation algorithms include ML and LS algorithms, where the ML algorithms have been applied to the above three scenarios but the LS algorithms have not been applied to the first scenario.

1) ML Algorithms in Direct Reception: As a seminal paper for an ML estimation in direct reception, Foschini et al. proposed an ML-based data reception scheme [60], where the PDF of the phase noise is approximated as a Tikhonov

PDF and the optimal ML data detector is designed in a high SNR range. Based on the results of [60], Pitarokoilis et al. proposed an ML-based data reception scheme in an uplink SIMO system [61], where different channels and phase-noise states are considered, including deterministic and known channels, stochastic and unknown channels, identical phase noise sources, and independent phase noise sources over different antennas. In [61], the optimal ML data detector under all channel and phase-noise states was also designed, where a closed-form expression of the SER floor in a high SNR range is derived.

2) ML Algorithms in Stepwise Reception: In the literature, two types of the pilot are considered, time-domain and frequency-domain pilots, where the frequency-domain pilot is also referred to as the Continuous Wave (CW) pilot [62]. Since the CW-based phase-noise estimation schemes do not require prior knowledge of the data symbols or even do not require time synchronization, it has better estimation performance than the other types of schemes in practice [62]. Lin et al. proposed an ML-based phase-noise estimation scheme in an optical communication system [63], where a CW pilot is used and two stages are considered. In the first stage, a CW pilot is extracted from the received signal via a digital Gaussian bandpass filter with a suitable order and bandwidth. In the second stage, phase noise is estimated using an ML estimator with the help of the CW pilot. Then, Zhang et al. provided a detailed description of the scheme [63] and presented corresponding theoretical analyses [64], where key parameters, i.e., pilot-to-signal ratio and bandwidth of the bandpass filter, are optimized. Based on [63], [64], Balogun et al. further proposed a joint estimation scheme for both phase-noise and CFO in a Coherent Optical-OFDM (CO-OFDM) system [65], where a CW pilot is used.

Specifically, since the CFO varies slowly, it is estimated first by an ML estimator. Then, the phase noise is further estimated, where the CFO is assumed to have been compensated for perfectly. Closed-form expressions of both the CFO estimate and the phase-noise estimate are derived to avoid exhaustive search. Unlike conventional schemes, where pilot subcarriers are predetermined, the scheme [65] adaptively defines the pilot subcarriers according to the phase of the data subcarriers.

In comparison with the results in [63], [64], [65], Gävert and Eriksson provided a more comprehensive theoretical analysis for the phase-noise estimation [62]. Specifically, Gävert and Eriksson analyzed several phase-noise estimation algorithms with a CW pilot, including the Best Linear Unbiased Estimator (BLUE), the ML estimator, and the approximate Minimum Variance Unbiased (MVU) estimator. The BLUE algorithm is non-optimal, the ML algorithm is optimal for a small phase noise but far away from optimal for a large phase noise, and the approximate MVU algorithm is nearly optimal. The approximate MVU algorithm always outperforms the BLUE algorithm and has lower complexity than the ML algorithm. However, as the phase noise increases, the approximate MVU algorithm cannot reach the corresponding Cramér-Rao Lower Bound (CRLB) since certain approximations in the derivation of the estimator are no longer valid.

In [62], [63], [64], [65], only phase-noise estimation is discussed. However, Combes and Yang discussed joint phase-noise estimation and data reception [66]. Specifically, Combes and Yang designed an approximate ML algorithm, called the Self Interference Whitening (SIW) algorithm, and then proposed a data reception scheme with the SIW algorithm in a Multiple-Input Multiple-Output (MIMO) system, which consists of two stages [66]. In the first stage, the phase noise is estimated first by an ML estimator. In the second stage, the data is whitened according to the estimated phase noise and then detected by an ML data detector, where the problem of minimum Euclidean distance detection is solved by a sphere decoder. The performance of the SIW algorithm is proven to be near the optimal ML data detector. However, in large-scale communication systems, the complexity of the SIW algorithm is very high due to the sphere decoder.

3) ML Algorithms in Decision-Feedback Reception: Kam proposed a data reception scheme [67], which consists of two stages. In the first stage, the phase noise is estimated first by the optimal ML estimator with the help of the estimated data symbols, where it is difficult to derive an explicit solution. Then, Kam provided an approximate ML estimator which can be implemented in practice. In the second stage, based on the estimated phase noise, coherent data reception is processed. Then, Kam et al. proposed a data reception scheme [68] to improve the Symbol Error Probability (SEP) of the scheme [67], which consists of two stages as well. In the first stage, the conditional PDF of the phase noise is estimated through an ML estimator with the help of the estimated data symbols. In the second stage, the optimal ML detector is used to detect data symbols. In [68], to save computational complexity, Kam et al. further proposed a practically feasible optimal ML detector, where the conditional PDF of the phase noise is approximated as the Tikhonov PDF.

Since, in the schemes [67], [68], the window length of the filter, i.e., the number of the estimated data symbols for phase-noise estimation, is fixed, Kam et al. proposed an adaptive data reception scheme with a variable window length of the filter to further improve the reception performance [69]. Specifically, Kam et al. proposed an adaptive ML algorithm for phase-noise estimation based on a first-order recursion technique, where the window length of the filter is adaptively calculated without prior knowledge of the phase noise.

Kam extended the scheme [67] to a coherent optical communication system [70], which has superior performance in the case that nonlinear phase noise dominates. Then, Kam et al. further extended the scheme [69] to a coherent optical communication system [71], where the problem of the fixed window length of the filter in [70] is addressed. In [72], Meiyappan et al. provided performance comparisons between the scheme [67] and the scheme [69], where only simulation results are provided and CFO estimation is not considered. Then, Meiyappan et al. proposed a data reception scheme [73], where a Complex-Weighted ML (CW-ML) algorithm is designed for both phase-noise and CFO estimation with the help of the estimated data symbols. The performance of the CW-ML algorithm in a low SNR range, which was neglected in [73], was studied in [74]. Meiyappan et al. further proposed two adaptive CW-ML algorithms [75], [76] to avoid the optimization of the window length of the filter. Specifically, a joint estimation for both the phase noise and CFO without considering a data reception is considered in [76], while a data reception scheme is further considered in [75].

4) LS Algorithms in Stepwise Reception: Casas et al. proposed an LS-based phase-noise estimation scheme in an OFDM system [77], where phase noise is modeled as a sum of discrete time-domain components. Leshem and Yemini proposed a phase-noise estimation scheme with an improved LS algorithm in an OFDM system [78], where CW pilots are used. In comparison with the scheme in [77], a more accurate model, i.e., the Karhunen-Loeve representation of the phase-noise covariance matrix, is used to describe a phase noise that is fairly stationary in a locked system [78]. Zou et al. proposed a data reception scheme in an OFDM system [79], which consists of two stages. In the first stage, pilot symbols are transmitted, and then both the channel response and phase noise are estimated at the receiver by iteratively solving an LS problem, where estimates of the channel response and phase noise are obtained separately. Note that, since the channel response is assumed to vary slowly, the channel estimates can be used for subsequent data reception. However, since the phase noise varies rapidly, the phase-noise estimates cannot be used for the data reception in the first stage. In the second stage, OFDM data symbols and pilot symbols are transmitted, and then the receiver estimates the phase noise and detects the data symbols according to the estimated channel response by iteratively solving an LS problem. The proposed scheme can effectively reduce the negative effect of an Inter Carrier Interference (ICI) due to the phase noise on the OFDM system, which significantly improves the reception performance.

Zou et al. considered a more complex scenario as compared to [79], i.e., a MIMO-OFDM system, where an LS algorithm is

used for joint estimation of the channel-response, phase-noise, and CFO [80]. Specifically, since each transmit and receive antenna is equipped with an independent oscillator [80], more phase-noise parameters need to be estimated. The scheme [80] consists of two stages: in the first stage, the channel response is estimated by an LS estimator; in the second stage, both the phase noise and CFO are estimated by an LS estimator and then the data symbols are detected by an LS detector. Ngebani et al. proposed a joint estimation scheme of both the channel response and phase noise with an LS algorithm in a MIMO-OFDM system [81], where a novel placement of pilots and nulls in the preamble and data frame of a MIMO-OFDM system is presented. Sokal et al. proposed a tensor decomposition-based data reception scheme in a frequency-selective MIMO-OFDM system [82], where the received samples are modeled by a third-order tensor model to separate the phase noise from the channel. The proposed scheme [82] consists of two stages. In the first stage, both the channel response and phase noise are jointly estimated with an LS estimator, which consists of two estimation algorithms. Specifically, the first algorithm is designed based on a bilinear alternating LS estimator, where two LS problems are solved in an alternating way, while the second one is designed based on the LS-Khatri-Rao factorization solution that solves multiple rank-one factorizations for joint channel-response and phase-noise estimation. In the second stage, data symbols are detected by a ZF equalizer. Xie et al. proposed a joint channel-response and phase-noise estimation scheme with an LS algorithm in an asynchronous MIMO system [83], where the CRLB is also derived. Specifically, the signals of each transmitter antenna are transmitted sequentially with an intentional timing offset, which effectively suppresses the negative effect of the Inter Antenna Interference (IAI) and improves the estimation performance in a synchronous MIMO system [38].

5) LS Algorithms in Decision-Feedback Reception: Mehrpouyan et al. proposed an LS estimator for joint channel-response and phase-noise estimation in a MIMO system with stepwise reception [38]. It also introduced a weighted LS-based phase-noise estimation scheme for decision-feedback reception, where the phase noise is estimated based on the estimated data symbols and the corresponding CRLBs are derived.

6) Lessons: We have learned the following lessons for classical phase-noise estimation schemes:
- Although the ML algorithm is generally treated as the optimal solution, it cannot be derived in a closed-form expression, or even practically useful since the cost function involves nonlinear variables with high complexity.
- For the low-complexity implementation of the ML algorithm, approximation methods are often used, such as by approximating a complicated PDF into a simpler and more manageable one, e.g., a normal distribution, or transforming Cartesian complex variables into polar representations.
- In comparison with the ML algorithm, LS algorithms are appealing alternatives in practice due to their closed-form expressions.

- LS algorithms are more often used in multi-parameter estimation, e.g., joint estimation of the channel-response and phase-noise, joint estimation of the channel-response, phase-noise, and CFO, and joint estimation of the phase noise and data symbols, whereas ML algorithms are usually used for single-parameter estimation.

C. Bayesian Estimation Schemes

For convenience, we use the term (L)MMSE to represent both MMSE and LMMSE algorithms without causing any confusion. Bayesian estimation algorithms include MAP and (L)MMSE algorithms. The MAP algorithms have been applied to direct, stepwise, and decision-feedback reception. However, the (L)MMSE algorithms have not been applied to the first type of reception.

1) MAP Algorithms in Direct Reception: Yang et al. proposed a MAP algorithm in an uplink massive MIMO system [84], where the phase noise is approximated as a zero-mean Gaussian RV. Specifically, the scheme [84] was designed based on approximate Bayesian inference, where the channel response is assumed to be known at the receiver.

2) MAP Algorithms in Stepwise Reception: Lin et al. proposed a MAP algorithm for the joint estimation of the channel response, phase noise, and CFO in an OFDM system [85]. In addition, a modified estimator based on a specific training symbol as well as a third estimator based on conjugate gradient iteration were further proposed for complexity reduction. In comparison with the schemes in [85], Tao et al. proposed an estimation scheme without requiring the prior knowledge of the channel length [86], which not only improves the estimation performance but also reduces the complexity. Specifically, the CFO is first estimated based on specially designed time-frequency pilot symbols. Then, using the estimated CFO, the phase noise and frequency-domain channel transfer function are further jointly estimated by a MAP estimator. Ishaque and Ascheid proposed a data reception scheme [87], which considered a more complex scenario as compared to the scheme [86], i.e., a MIMO-OFDM system. Specifically, a MAP estimator for phase noise is derived under the assumption that perfect channel knowledge is available. Since the MAP estimator is near-optimal but highly complex due to the inversions of a high-dimension matrix, a sub-optimal extension of the MAP estimator with lower complexity is further proposed and analyzed.

3) MAP Algorithms in Decision-Feedback Reception: Wang et al. proposed both an ML algorithm for joint carrier frequency and initial carrier phase estimation and a MAP-based phase-noise estimation scheme in an AWGN channel [88]. Specifically, an unknown but deterministic carrier frequency and initial phase are estimated by the ML estimator, while an unknown and random phase noise is estimated by a MAP estimator, where both the ML and MAP estimators are implemented iteratively with the help of the estimated data symbols. The CRLB for carrier-frequency and initial carrier-phase estimation as well as a Bayesian CRLB (BCRLB) for phase-noise estimation are derived in [88]. Wang et al. proposed a data reception scheme in an OFDM relay

system [89], which consists of two stages. Specifically, in the first stage, the channel response, phase noise, and CFO are jointly estimated by a MAP estimator with the help of pilot symbols, where the correlation between phase-noise parameters is utilized to reduce the complexity and overhead. In the second stage, based on the above estimation results, the phase noise is tracked and data symbols are detected by a MAP estimator with the help of estimated data symbols, where the Hybrid CRLB (HCRLB) is derived. Nasir et al. proposed a MAP-based phase-noise estimation scheme [90], where the phase noise is estimated by using both the estimated data symbols and estimated phase noise, and a soft-decision algorithm is considered.

Note that it is challenging for a MAP algorithm to be implemented in practice due to its high complexity. Thus, the objective of the following schemes is to reduce the complexity and ensure a performance close to that of the MAP algorithm. Krishnan et al. proposed three approximate MAP data-reception schemes in a MIMO system over a quasi-static fading channel [91], including the Sum-Product (SP) algorithm, Smoother-Detector Structure (SDS) algorithm, and Variational Bayesian (VB) algorithm, all of which consist of two stages. In the first stage, both the channel response and phase noise are jointly estimated by an LS estimator, similar to [38]. In the second stage, both the phase noise and data symbols are jointly estimated by the above approximate MAP algorithms with the help of estimated data symbols. Shayovitz and Raphaeli proposed a data reception scheme in a strong phase-noise AWGN channel [92], where an SP algorithm is used for both phase-noise estimation and data reception with the help of estimated data symbols. Specifically, a novel approximation of the SP algorithm by exploiting Tikhonov mixtures canonical models is studied, and a data reception scheme is further proposed to save the complexity. Note that the SP-like algorithm, which utilizes factor graphs, is also referred to as the Message Passing (MP) algorithm. Then, many enhanced versions of the MP algorithm with the help of estimated data symbols have been proposed, e.g., the Belief Propagation (BP) algorithm, Variational Message Passing (VMP) algorithm, also referred to as the Mean Field (MF) algorithm, and Generalized Approximate Message Passing (GAMP) algorithm. Shi et al. proposed a data reception scheme in an OFDM system combined with Index Modulation (OFDM-IM) [93], where a GAMP algorithm is used for both phase-noise estimation and data reception. Specifically, two phase-noise models are considered, i.e., the Wiener model and the truncated discrete cosine transform expansion model, where the channel response is assumed to be perfectly known at the receiver in theoretical analyses, while the effects of imperfect channel estimation are considered in the simulation results.

It is possible to improve the estimation performance by combining multiple estimation algorithms. Wang et al. proposed a hybrid data reception scheme [94], where the BP algorithm, MF algorithm, and Expectation Propagation (EP) algorithm are jointly exploited. Specifically, a linear phase noise is handled by both the BP and EP algorithms, whereas a nonlinear phase noise is handled by the MF algorithm, where

the channel response is assumed to be known at the receiver. In the MF algorithm, a second-order Taylor approximation is proposed to achieve Gaussian approximation for saving the complexity. Wang et al. further proposed a hybrid data reception scheme [95], where both the SP algorithm and the VMP algorithm are jointly exploited. Specifically, the VMP algorithm is used to deal with the observation factors, while the SP algorithm is used to deal with the remaining factors.

4) (L)MMSE Algorithms in Stepwise Reception: Le and Nguyen proposed a data-reception scheme in a Circular Filter-Bank Multi-Carrier Offset Quadrature Amplitude Modulation (CFBMC-OQAM) system [96], which consists of two stages. Specifically, in the first stage, the channel response is estimated by an MMSE estimator with the help of pilot symbols. Then, based on the estimated channel response, the phase noise is further estimated by an MMSE estimator. In the second stage, the data symbols are detected after the phase noise of the data symbols is compensated for based on the estimated phase noise obtained in the first stage. Chung et al. proposed a joint LMMSE estimation scheme for both the channel response and phase noise in an mm-Wave system [97]. Specifically, a phase-noise spectrum approximation method is designed and the coherence-bandwidth feature of an mm-Wave system is extracted to reformulate the under-determined system into a fully determined system, where an LMMSE estimator for joint estimation of the channel-response and phase-noise is derived. Moreover, in [97], the real implementation of the proposed LMMSE estimator is further studied, where a matrix inversion is avoided to save the complexity.

5) (L)MMSE Algorithms in Decision-Feedback Reception: Petrovic et al. proposed a phase-noise estimation scheme in an OFDM system [98], where higher spectral components of the phase noise can be estimated by an MMSE estimator with the help of the estimated data symbols. Then, they proposed a phase-noise estimation scheme with an MMSE algorithm in an OFDM system [3], which improves the estimation performance of the scheme [98]. Specifically, they expanded the phase-noise model of an FR oscillator in [98] by simultaneously considering both the phase-noise model of the FR oscillator and the PLL circuit, where the variance of an ICI as a function of the phase noise is also derived. Bittner et al. extended the scheme [3] to a MIMO-OFDM system [99], where both phase-noise estimation and data reception are considered. Specifically, first, they revealed the fact that an ICI caused by the phase noise, which does not follow a Gaussian distribution, can be typically approximated by a truncated Fourier series. Then, they proposed a data reception scheme, which consists of two stages. In the first stage, the phase noise is estimated by an LMMSE estimator that is assisted by the output of the decoder. In the second stage, data symbols are detected by an LMMSE detector. Since the approximation of the ICI in [99] is not sufficiently accurate, the authors further proposed a data reception scheme in a MIMO-OFDM system [100], where the ICI is approximated by a Laplace distribution. Mehrpouyan et al. proposed a phase-noise estimation scheme with an Extended Kalman Filter (EKF) algorithm to reduce the system overhead and delay [38], where the phase noise is estimated based on both the estimated

data symbols and estimated phase noise via stepwise reception. Although the Kalman filter is the optimal MMSE estimator, the EKF algorithm suffers from performance loss [101]. This is because the phase noise is not linearly related to the received signals, the EKF algorithm requires linearization of the received signals, which introduces a loss of estimation performance. Nasir et al. further proposed an improved EKF for phase-noise estimation and derived the corresponding BCLRBs in [90]. Specifically, they adopted a soft-decision algorithm rather than a hard-decision algorithm used in [38], which not only improves the estimation performance but also reduces the pilot overhead.

Salim et al. proposed a data reception scheme with the Expectation Conditional Maximization (ECM) algorithm in an OFDM system [102], which consists of three stages. In the first stage, the phase noise is estimated by an EKF along with OFDM pilot symbols. In the second stage, both the channel response and CFO are estimated by minimizing a negative log-likelihood function. In the third stage, with the help of the estimated data symbols, the phase noise is tracked by the EKF, and data symbols are detected based on the estimated channel response, phase noise, and CFO. The corresponding HCRLB is also derived.

6) Lessons: We have reviewed some representative Bayesian-estimation based data reception schemes, which leads to the following lessons:

- A MAP algorithm has a similar structure to an ML algorithm with some modifications.
- For the MAP-based data reception schemes in direct reception, it is usually assumed that 1) channel responses are pre-estimated; or 2) perfect channel knowledge is available at the receiver.
- In the presence of phase noise, the posterior PDF of the received symbols can only be obtained after the phase-noise marginalization. For a Wiener phase-noise process, the posterior PDF of phase noise is analytically intractable, which makes the corresponding MAP algorithms unimplementable.

D. Machine-Learning Based Estimation Algorithms

In the literature, DL algorithms have been employed only with stepwise reception.

1) DL Algorithms in Stepwise Reception: Mohammadian et al. proposed a data reception scheme for both OFDM and Generalized Frequency Division Multiplexing (GFDM) systems [103], where a Deep Neural Network (DNN) and a Convolutional Neural Network (CNN) are used for phase-noise estimation and data detection, respectively, as illustrated in Fig. 13. Specifically, an LS estimator is first used to estimate the channel response. Then, the phase-noise estimation is designed based on a full-connected DNN, while the data detection is designed based on a fully connected CNN. Mattu and Chockalingam proposed a joint channel-response and phase-noise estimation scheme in an OFDM system over a phase-noise contaminated doubly-selective fading channel [104], which eliminates the static channel assumption in [103] and works well in time-varying channel conditions.
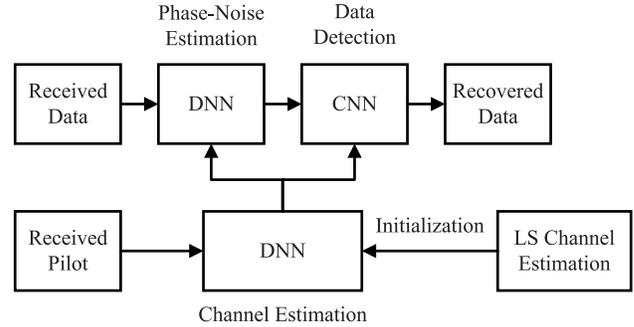

Fig. 13.   Block diagram of a DL-based data reception scheme [103].

Specifically, the channel-response estimation problem on a doubly-selective time-frequency grid is shown to be equivalent to an image completion problem based on sparse data. The phase noise is further estimated according to both the estimated channel response and pilot symbols.

2) Lessons: Some lessons on machine-learning based data reception schemes are the following.

- The advantages of DL algorithms are twofold. First, they eliminate the iterative process in both the classical and Bayesian estimation algorithms. Second, they can significantly improve the MSE performance without requiring extra statistical information on the parameters to be estimated.
- DL algorithms also have the following drawbacks. First, it is challenging to derive the association of different parameters and even to optimize a particular parameter. Second, DL algorithms may suffer from overfitting since the model obtained based on the training samples may not be suitable for other samples.

E. Summary

We summarize the above phase-noise estimation and data reception schemes as well as related limitations and issues in Table V. There are three categories of algorithms for phase-noise estimation: 1) classical estimation algorithms, represented by the ML and LS algorithms; 2) Bayesian estimation algorithms, represented by the MMSE, LMMSE, and MAP algorithms; and 3) machine-learning based estimation algorithms, represented by the DL algorithms. Both classical and machine-learning based estimation algorithms do not need prior knowledge of the phase noise, whereas the Bayesian estimation algorithms do. Although machine-learning based estimation algorithms provide better estimation performance than the remaining algorithms, they cannot provide an explicit relationship between the system model and parameters of the phase noise and may suffer from the overfitting problem. There are also three types of data reception schemes: direct reception, stepwise reception, and decision-feedback reception. Both classical and Bayesian estimation algorithms can be applied to the above three types of reception, whereas the machine-learning based estimation algorithms have only been applied to the second type.

TABLE V
PHASE-NOISE ESTIMATION ALGORITHMS IN VARIOUS DATA RECEPTION SCHEMES

Authors	Estimation Algorithms	Reception Types	Contributions	Open Issues
Foschini et al. [60] and Pitarokoilis et al. [61]	ML	Direct Reception	ML-based Data reception schemes [60], [61]	High computational overhead and high latency
Zhang et al. [64], Lin et al. [63], Combes and Yang [66], Balogun et al. [65], and Gävert and Eriksson [62]	ML	Stepwise Reception	1) Analysis of three phase-noise estimation algorithms, including the BLUE, the ML estimator, and the approximate MVU estimator [62] 2) Phase-noise estimation schemes with an ML algorithm [63], [64] for an optical communication system 3) An ML-based joint phase noise and CFO estimation scheme [65] for a CO-OFDM system 4) A data reception scheme with the SIW algorithm [66] for a MIMO system	High pilot overhead
Kam [67], Kam et al. [68], Kam et al. [69], Zhang et al. [71], Zhang et al. [70], Meiyappan et al. [72], Meiyappan et al. [73], Meiyappan et al. [74], Meiyappan et al. [76], Meiyappan et al. [75], and Wang et al. [88]	ML	Decision-Feedback Reception	1) An adaptive phase-noise and CFO estimation scheme with the CW-ML algorithm [76] 2) An ML algorithm for joint carrier frequency and initial carrier phase estimation [88] in an AWGN channel 3) ML-based data reception schemes [67], [68] 4) An adaptive CW-ML based data reception scheme [75] 5) Data reception schemes with a) an adaptive ML algorithm [[69]–[71]; and b) the CW-ML algorithm [73], [74] 6) Simulation comparisons [72] between the scheme [69] and the scheme [67]	High computational overhead and high latency
Casas et al. [77], Zou et al. [79], Mehrpouyan et al. [38], Huang et al. [80], Leshem and Yemini [78], Ngebani et al. [81], Sokal et al. [82], and Xie et al. [83]	LS	Stepwise Reception	1) Phase-noise estimation schemes with a) an LS algorithm [79]; and b) an improved LS algorithm [78] for an OFDM system 2) LS-based joint channel response and phase noise estimation schemes for a) a MIMO system [38]; b) a MIMO-OFDM system [81]; and c) an asynchronous MIMO system [83] 3) LS-based data reception schemes for a) an OFDM system [77]; and b) a MIMO-OFDM system [80] 4) A tensor decomposition-based LS data reception scheme for a frequency-selective MIMO-OFDM system [82]	High pilot overhead
Mehrpouyan et al. [38]	LS	Decision-Feedback Reception	An LS-based phase-noise estimation scheme for a MIMO system	High pilot overhead, high computational overhead, high latency, and strong synchronization assumption
Yang et al. [84]	MAP	Direct Reception	A MAP-based data reception scheme for an uplink massive MIMO system	High computational overhead and high latency
Lin et al. [85], Tao et al. [86], and Ishaque and Ascheid [87]	MAP	Stepwise Reception	1) MAP-based joint channel response, phase noise, and CFO estimation schemes [85], [86] for an OFDM system 2) A MAP-based data reception scheme [87] for a MIMO-OFDM system	High computational overhead and high latency
Nasir et al. [90], Krishnan et al. [91], Shayovitz and Raphaeli [92] Wang et al. [89], Wang et al. [94], Wang et al. [95], Yang et al. [84], Shi et al. [93], and Wang et al. [88]	MAP	Decision-Feedback Reception	1) MAP-based phase-noise estimation schemes a) for a MIMO system [90]; and b) in an AWGN channel [88] 2) MAP-based data reception schemes for a) an OFDM relay system [89]; and b) an uplink massive MIMO system [84] 3) Data reception schemes with a) an SP algorithm [91], [92]; b) an SDS algorithm [91]; c) the VB algorithm [91]; d) the BP-MF-EP algorithm [94]; e) the SP-VMP algorithm [95]; and f) a GAMP algorithm [93]	High computational overhead and high latency
Le and Nguyen [96], and Chung et al. [97]	(L)MMSE	Stepwise Reception	1) An MMSE-based data reception scheme [96] for a CFBMC-OQAM system 2) An LMMSE-based data reception scheme [97] for an mm-Wave system	High pilot overhead, high computational overhead, and high latency
Petrovic et al. [98], Petrovic et al. [3], Bittner et al. [99], Bittner et al. [100], Mehrpouyan et al. [38], Nasir et al. [90], and Salim et al. [102]	(L)MMSE	Decision-Feedback Reception	1) Phase-noise estimation schemes with a) an MMSE algorithm [3], [98]; and b) an EKF algorithm [38], [90] for an OFDM system 2) Data reception schemes with a) an LMMSE algorithm [99], [100]; and b) an ECM algorithm [102]	Strong synchronization assumption
Mohammadian et al. [103], and Mattu and Chockalingam [104]	DL	Stepwise Reception	1) A DL-based joint channel response and phase noise estimation [103] for an OFDM system over a phase-noise doubly-selective fading channel 2) A DL-based data reception scheme [104] for OFDM and GFDM systems	High computational overhead and high latency

V. JOINT TRANSMIT AND RECEIVE DESIGNS WITH PHASE-NOISE MITIGATION

In the previous section, phase-noise estimation and mitigation schemes were designed by leveraging the resources of the receiver, whereas, in this section, the problem is addressed by utilizing the resources of both the transmitter and receiver, hence referred to as the joint transmit and receive designs.
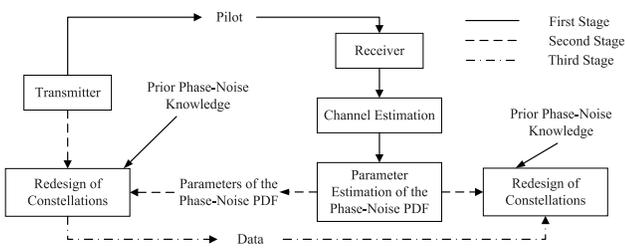
Fig. 14. Overview of joint transmit and receive designs with phase-noise mitigation, where the constellation is jointly optimized.

Fig. 15. Optimization examples of a joint transmit and receive design in a 16-point circular QAM constellation system [108].

A. Overview

To improve the performance of phase-noise estimation and data reception schemes described in the previous section, various joint transmit and receive designs were designed, where the transmit constellation and the receiver are jointly optimized. A joint transmit and receive design often consists of three stages, as illustrated in Fig. 14. In the first stage, the transmitter sends a pilot for channel and phase-noise estimation, where the specific form of the phase-noise PDF is available but its parameters are unknown. In the second stage, the receiver redesigns the constellation according to the estimated results, and the guidelines of the constellation design are agreed upon with the transmitter in advance. Meanwhile, since the receiver sends the estimated parameters of the phase-noise PDF to the transmitter, the transmitter obtains the same constellation as the receiver. In the third stage, the transmitter sends a data frame based on the redesigned constellation, where the remaining processes are similar to those in the data reception schemes described in Section IV. Note that there are two approaches for the constellation optimization. The first approach is that the redesigned constellation should satisfy some certain shape, e.g., the shape of a square QAM constellation, circular QAM constellation, or spiral QAM constellation, whereas the second approach relaxes this constraint. Although the second approach enjoys better reception performance than the first one, it suffers from higher complexity [105].

B. Joint Transmit and Receive Designs With the First Approach

1) Square QAM Constellation: In a square QAM constellation system, since a phase noise rotates the constellation points, an effective countermeasure is to rotate the constellation points at a transmitter in advance to cancel the negative effect of the phase noise on a receiver. Khorov et al. proposed a joint transmit and receive design with the square QAM constellation by rotating the constellation points of the secondary frame in a Non-Orthogonal Multiple Access (NOMA) Wi-Fi system [106], where the constellation points of the primary frame are unchanged. Since only simulation results and intuitive conclusions were provided in [106], they further provided performance analyses in [107], where the expression of the optimal rotation angle based on minimizing the BER is derived.

2) Circular QAM Constellations: Two factors should be considered in optimizing the constellation points in a circular
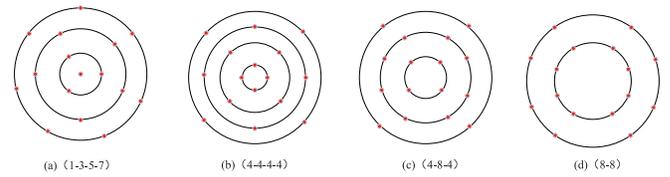
QAM constellation system: the number of points in each ring and the radius of each ring, as illustrated in Fig. 15. Yang et al. proposed a joint transmit and receive design with the circular QAM constellation by maximizing the Minimum Euclidean Distance (MED) between the constellation points under an average power constraint [105]. Moreover, an easy-to-implement circular constellation demodulation method was proposed as well [105]. Beygi et al. further improved the scheme [105] in a 16-point circular QAM constellation system [108], where an SER expression is derived and the constellation points are optimized by minimizing the SER. Specifically, the optimal radiuses for four constellations are derived by minimizing the SER, as illustrated in Fig. 15. Tariq et al. proposed a joint transmit and receive design in a 32-point circular QAM constellation system [109], where a closed-form approximate union bound expression for the SEP is derived. Specifically, they verified that the performance of a circular QAM constellation system is better than that of a square QAM constellation system in the presence of time-varying phase noise through simulations. Häger et al. proposed a joint transmit and receive design in a coherent optical communication system with the circular Amplitude Phase Shift Keying (APSK) constellation [110], which compensates for the effect of a nonlinear phase noise caused by the Kerr effect and saves the bandwidth resource as compared with the scheme [108]. Specifically, the PDF of the multi-level constellation is derived and the circular APSK constellation is optimized by minimizing the SEP.

3) Spiral QAM Constellations: In comparison with the traditional QAM constellation, the spiral QAM constellation has a lower symbol density and better robustness to phase noise. Kwak et al. proposed a joint transmit and receive design in a spiral QAM constellation system [111]. Specifically, a construction method of the spiral QAM constellation is proposed, but how to find the optimal symbol mapping rule is not discussed, which is very difficult. Ugolini et al. proposed a new construction method for a spiral QAM constellation system [112], where the constellation points are arranged along an Archimedean spiral and the radial distance between their spiral laps is constant. In comparison with the scheme [111] and other multi-level constellation systems, such as APSK, where multiple parameters should be optimized, e.g., the number of rings, the number of constellation points on each ring, and the radius of the ring, the scheme of [112] has lower complexity because there is only one parameter to be optimized. Moreover, it can be adjusted according to the phase noise condition, which is suitable for the current state-of-the-art coding systems without redesigning the code.

TABLE VI
CONSTELLATION OPTIMIZATION IN VARIOUS JOINT TRANSMIT AND RECEIVE DESIGNS

| Authors	Optimized Constellation	Channel	PN Model	Contributions	Open Issues
Khorov et al. [106] and Khorov et al. [107]	Square QAM Constellation	Not Mentioned [106] AWGN [107]	Not Mentioned [106] Tikhonov PDF [107]	Joint transmit and receive designs with the square QAM constellation in a NOMA Wi-Fi system [106], [107]	Optimization based on other metrics
Yang et al. [105], Beygi et al. [108], Häger et al. [110], and Tariq et al. [109]	Circular QAM Constellations	AWGN [105], [109] Fiber-Optical Channel [108] Coherent Optical Channel [110]	Constant [105] Nonlinear [110], [110] Gaussian PDF [109]	1) Joint transmit and receive designs with the circular QAM constellation by a) maximizing the MED [105]; b) minimizing the SER [108]; and c) minimizing the SEP [110] 2) Closed-form approximate union bound expression for the SEP of a 32-point circular QAM constellation system [109]	Optimization based on other metrics
Kwak et al. [111], and Ugolini et al. [112]	Spiral QAM Constellations	AWGN	Gaussian PDF	Joint transmit and receive designs with the spiral constellation [105], [108]	Optimization based on other metrics
Kayhan and Montorsi [113], and Krishnan et al. [114]	No Constraint on Constellation Shape	AWGN	Tikhonov PDF [113] Gaussian PDF [114]	1) A joint transmit and receive design with the second approach by maximizing the AMI under the average power constraint [113] 2) A joint transmit and receive design [114] including three optimization algorithms based on different metrics in different application scenarios	High computational complexity
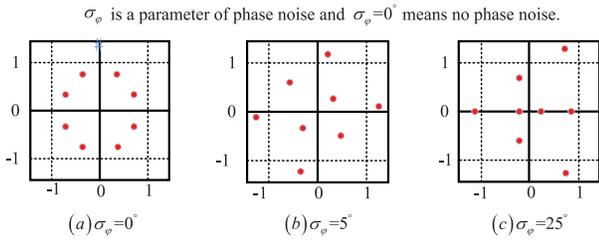
Fig. 16. Optimization examples of a joint transmit and receive design with the second approach [113].

4) Lessons: We have reviewed several joint transmit and receive designs with the first approach, which leads to the following lessons:

- Both the circular QAM constellation and spiral QAM constellation have better performance than the square QAM constellation, at the cost of higher optimization complexity.
- Performance and complexity comparisons between the circular QAM constellation and spiral QAM constellation are not yet available.

C. Joint Transmit and Receive Designs With the Second Approach

1) No Constraint on Constellation Shape: Kayhan and Montorsi proposed a joint transmit and receive design with the second approach [113], where the Achievable Mutual Information (AMI) between the transmitted signal and the received signal is approximated and the Simulated Annealing (SA) algorithm is used to maximize the AMI for optimizing the constellation under the average power constraint. Since the SA algorithm is relatively slow for higher-order constellation systems, they mainly analyzed an 8-point constellation system, as illustrated in Fig. 16, where σ_φ is a parameter of phase noise and $\sigma_\varphi = 0$ means no phase noise. Since there is no regularity when optimizing the constellation as the value of

σ_φ changes, the complexity of optimizing the constellation with the second approach is higher as compared to that with the first approach. Krishnan et al. proposed a joint transmit and receive design with the second approach [114], where three optimization algorithms are proposed based on different metrics, i.e., SEP and mutual information, in different application scenarios.

2) Lessons: Regarding joint transmit and receive designs with the second approach, we have the following lessons:

- In comparison with the first approach, the schemes with the second approach have a significant performance gain, but they suffer from higher complexity.
- Since the constellation optimization in the schemes with the second approach is not regular, the optimal solutions obtained based on different metrics, e.g., SEP and mutual information, are different.

D. Summary

The contributions and concepts of joint transmit and receive designs and related limitations and issues are listed in Table VI. There are two approaches for the constellation optimization in joint transmit and receive designs with phase-noise mitigation, where the first approach considers a specific constellation shape, e.g., square QAM constellation, circular QAM constellation, and spiral QAM constellation, and the second approach relaxes this constraint. The schemes with the second approach have better reception performance than those with the first approach, but they suffer from higher complexity.

VI. APPLICATIONS OF PHASE NOISE IN WIRELESS COMMUNICATIONS

Phase noise, traditionally viewed as a detrimental factor in wireless communication systems, can also present unique opportunities for enhancing system performance and security. This section explores various applications of phase noise in wireless communications, highlighting its beneficial uses
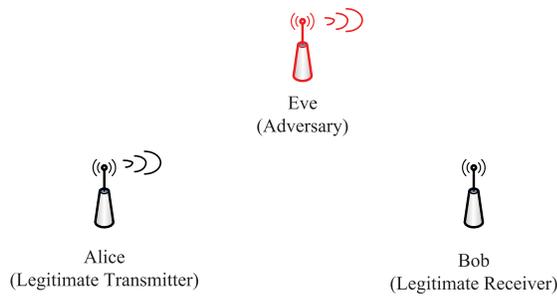
Fig. 17.   Typical threat model in the PLA with three nodes.

Fig. 18.   Flow diagram of a phase-noise based PLA scheme.

across different scenarios. First, we examine how phase noise can be utilized for PLA, which is aimed at enhancing the security of communication systems. Next, we highlight the benefits of phase noise in transmitter identification, where it improves the ability to differentiate between different signal sources based on unique noise characteristics. Additionally, we discuss its role in generating cryptographically secure random numbers, a crucial element for encryption and security protocols. Finally, we investigate the application of phase noise in protecting transmitted signals from eavesdropping, enhancing the security of wireless communications.

A. Phase-Noise Based PLA

1) Overview: In wireless communications, authentication is a key concern since the open nature of a wireless channel introduces security vulnerabilities, e.g., impersonation attacks and substation attacks. Recently, PLA has attracted a lot of attention because of its advantages, i.e., information-theoretic security, low complexity, and high compatibility [115]. Even for two devices produced by the same manufacturer, the device-based features are different due to production imperfection, e.g., phase noise, which cannot be controlled by enhancing the computational power of the adversary [10], [11]. We illustrate the typical threat model in PLA with three nodes in Fig. 17.

- Alice is a legitimate transmitter, who sends a legitimate signal to Bob.
- Bob is a legitimate receiver, who authenticates the origin of the received signal using a certain PLA scheme.
- Eve is an adversary, who impersonates Alice to send forged signals to Bob.

A phase-noise based PLA scheme consists of two stages: a training stage and a message-transmission stage, as illustrated in Fig. 18. Note that the security level of the training stage is ensured through an upper-layer security mechanism.

2) Phase-Noise Based PLA Schemes: Zhao et al. proposed a PLA scheme based on the features of both the phase noise and clock [12], where a simple multiple kernel learning algorithm is used to classify the features. Zhang et al. proposed a PLA scheme based on the features of both the channel response and phase noise [11], where an ML estimator and an EKF are used to estimate the channel response and phase noise, respectively. Moreover, they analyzed three properties in terms of covertness, robustness, and security, where closed-form expressions for the false alarm and detection
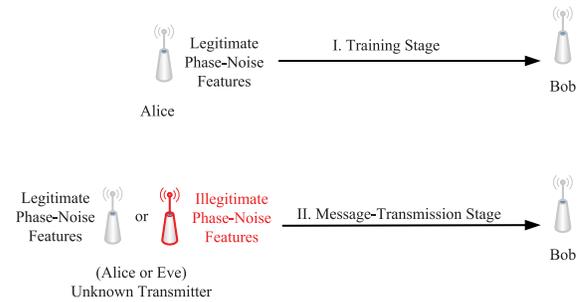
probabilities considering the quantization error are derived. Considering both impersonation and replay attacks, Zhang et al. further proposed a PLA scheme based on the features of both the channel response and phase noise in an uplink massive MIMO system [116], which improves the authentication performance of [11]. Specifically, an LMMSE estimator and an EKF are used to estimate the channel response and phase noise, respectively, where the estimation error covariance matrix of the channel response and phase noise is derived. Based on phase noise features, Xie et al. proposed two PLA schemes without using any quantization algorithm [10], including the Multiple Phase Noises PLA (MPP) scheme and the Enhanced Multiple Phase Noises PLA (EMPP) scheme. Specifically, these schemes introduce an artificial random phase to the transmitted symbols, which improves the authentication performance of [11]. Moreover, the corresponding optimal threshold is derived in closed form, which provides insight into the feasibility of these schemes, whereas [11] finds the optimal threshold via an exhaustive search method.

We take the PLA scheme [10] as an example to demonstrate the basic steps of a phase-noise based PLA scheme, which are presented in **Algorithm 1**.

3) Lessons: By reviewing the phase-noise based PLA schemes, we draw the following lessons:

- The phase-noise based PLA schemes enjoy high security, since the device-based features are different due to production imperfection, even with devices produced by the same manufacturer.
- If a phase-noise based PLA scheme uses a quantizer to make an authentication decision, it has the following limitations. First, it suffers from quantization errors. Second, it is challenging to determine a closed-form expression of the optimal threshold, which needs to be determined by an exhaustive search that may converge to a local optimum. Third, it is difficult to obtain an insight into the feasibility of the adopted PLA scheme, which suffers from changes in the communication environment.
- If a PLA scheme only uses phase noise as a feature for authentication, it is vulnerable to replay attacks. Thus, a wise solution is to utilize multiple features for authentication.
- The estimation of phase noise is quite complex, which leads to high computational load and communication latency in the phase-noise based PLA schemes.

Algorithm 1: Detailed Steps of the Phase-Noise Based PLA Scheme [10]

Training Stage: $(i-1)$ th **timeslot**

Step 1: Alice sends a request to Bob over a wireless channel.

Step 2: Bob receives the request.

Step 3: Bob first checks whether the request is legitimate through a certain upper-layer authentication mechanism. Then, if it is legitimate, Bob extracts the phase-noise features from the received signal to construct a white list; otherwise, Bob ignores the request.

Step 4: Bob sends an acknowledgement to Alice for preparing a message transmission.

Message-Transmission Stage: i th **timeslot**

Step 1: An unknown transmitter sends a message to Bob over the same wireless channel as that in the training stage.

Step 2: Bob receives a signal transmitted from an unknown transmitter.

Step 3: Bob extracts the phase-noise features from the current received signal and compares them with the corresponding ones of the white list. If the features belong to the white list, Bob accepts the received signal; otherwise, Bob rejects it.

Algorithm 2: Detailed Steps of the Phase-Noise Based Transmitter Identification Scheme [13]

Phase Noise Profiling Stage:

Step 1: Configure the signal generator to produce a CW signal at a specified frequency and connect it to a phase noise analyzer.

Step 2: Measure the phase noise of the transmitted CW signal, focusing on spurious content, noise floor amplitude, and noise curvature.

Step 3: Record the phase noise data for different transmitters under identical conditions.

Step 4: Compare the phase noise profiles, identifying unique characteristics, and catalog distinctive phase noise signatures for each transmitter.

Identification Stage:

Step 1: Measure the phase noise characteristics of a signal from an unknown transmitter.

Step 2: Compare the measured phase noise profile with cataloged signatures to identify the transmitter.

B. Phase-Noise for Transmitter Identification

1) Overview: While both phase-noise based PLA and phase-noise based transmitter identification use the device-specific nature of phase noise, they serve different purposes. PLA leverages phase noise for authenticating communication signals to ensure only legitimate users can communicate. In contrast, phase-noise based transmitter identification distinguishes transmitters, essential for identifying and managing signal sources in a network. PLA focuses on security, whereas transmitter identification aims to recognize and catalog legitimate transmitters based on their hardware-induced phase noise signatures.

In wireless communications, accurately identifying the source of a transmitted signal can be crucial for applications such as security, regulatory compliance, and interference management. Traditional identification methods often require complex processing and high-speed digitization. Phase noise offers a simpler, more passive alternative by leveraging inherent differences in the hardware components of transmitters. Phase noise characteristics, such as spurious content and noise floor amplitude, vary with the hardware configuration of the transmitter, making it possible to distinguish between different transmitters based on these measurements.

2) Phase-Noise Based Transmitter Identification Scheme: Cutitta and Dietlein proposed using phase noise measurements to differentiate transmitters, leveraging the fact that phase noise properties are influenced by the specific hardware components used in the transmitter [13]. The study involved comparing the phase noise of CW signals generated by different equipment. These measurements revealed distinct phase noise characteristics for each device, attributed to differences in their ROs, power supplies, and other components. They demonstrated that even transmitters producing nominally identical signals could be distinguished based on phase noise measurements. This method does not require prior knowledge of the transmitted signal's modulation or format, making it a practical and cost-effective solution for transmitter identification.

We use the transmitter identification scheme from [13] as an example to demonstrate the basic steps of a phase-noise based transmitter identification scheme, as presented in **Algorithm 2**.

3) Lessons: By reviewing the phase-noise based transmitter identification scheme, we draw the following lessons:

- This approach is passive and does not require high-speed digitization or detailed knowledge of the transmitted signal.
- Accurate phase noise measurement requires precise equipment and an understanding of how different hardware components influence the noise properties.
- The effectiveness of this method depends on the stability and repeatability of the phase noise characteristics over time and under varying operational conditions.

C. Phase-Noise Based Random Number Generation

1) Overview: True random number generators (TRNGs) leveraging phase noise are pivotal in cryptographic systems

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Algorithm 3: Detailed Steps of the Phase-Noise Based TRNG Scheme [14]

Initialization Stage:

Step 1: Initialize the ring oscillator circuit with a predefined configuration.

Step 2: Enable the ring oscillator and allow it to run freely to generate an oscillating signal.

Step 3: Measure the phase noise characteristics of the oscillator, considering multiple noise sources like white noise, flicker noise, and random walk noise.

Step 4: Calibrate the system by recording the phase noise over a set period to create a baseline noise profile.

Entropy Collection Stage:

Step 1: Continuously sample the output of the ring oscillator to capture the phase variations.

Step 2: Use a high-resolution time-to-digital converter to digitize the phase noise measurements.

Step 3: Analyze the sampled data to estimate the entropy, ensuring the randomness meets the TRNG requirements.

Step 4: Compare the real-time phase noise data with the baseline profile to detect and compensate for any drift or deviations.

Output Stage:

Step 1: Generate random numbers based on the digitized and processed phase noise data.

Step 2: Perform post-processing to ensure the statistical properties of the generated random numbers meet the desired cryptographic standards.

Step 3: Output the random numbers for use in cryptographic applications, ensuring continuous monitoring and validation of the noise characteristics.

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where unpredictability and security are paramount. These TRNGs exploit the randomness inherent in electronic noise within oscillator circuits, a feature vital for generating cryptographic keys and ensuring secure communications. The significance of phase noise-based TRNGs extends beyond typical cryptographic applications, impacting areas like secure IoT device functioning and blockchain technology, where the integrity and security of transactions rely heavily on robust random number generation.

2) Phase-Noise Based Random Number Generation Schemes: In the realm of TRNG development, Sunar et al. pioneered with their design that introduced robustness against active attacks, making significant strides in securing TRNGs under adversarial conditions [117]. Their TRNG employs a basic entropy source that undergoes robust conditioning to ensure the randomness meets high-security standards. However, the added security features increase design complexity and potential cost, posing challenges for resource-constrained environments. Later, Liu et al. advanced the field by designing a low-cost, power-efficient ring oscillator-based TRNG for smart card encryption [118]. The proposed TRNG utilizes a tetrahedral oscillator with enhanced jitter to improve randomness, addressing power efficiency and area utilization, which are critical in embedded systems. The oscillator sampling technique and a post-digital processor further enhance the output randomness. However, the design's large physical size and limited output rate may not meet all application demands. More recently, Peetermans and Verbauwhede provided a comprehensive analysis of oscillator phase noise characterized by various noise sources, such as thermal and flicker noise [14]. This analysis enhances understanding of how different types of noise impact randomness in TRNGs, particularly in Application-Specific Integrated Circuits (ASIC) implementations. Their work emphasizes the critical nature of noise management

in maintaining TRNG reliability and effectiveness. However, further research is needed to integrate and optimize these findings in practical applications.

We use the TRNG scheme from [14] as an example to demonstrate the basic steps of a phase-noise-based TRNG scheme, as presented in **Algorithm 3**.

3) Lessons: By reviewing the phase-noise-based random number generation schemes, we draw the following lessons:

- High accuracy in phase noise estimation is essential when designing a phase-noise-based TRNG. If the randomness properties e.g., distribution, spectral properties, of the phase noise are underestimated or misestimated, the generated random numbers may become predictable or biased, reducing the overall system security.

- Balancing enhanced security features with practicality in TRNG design is crucial. While early designs prioritized randomness, later works demonstrate a concerted effort to integrate security features that protect against both passive and active threats without significantly impacting power consumption or speed.

- The work by Peetermans and Verbauwhede [14] underscores the importance of considering long-term dependencies and noise behavior over extended periods. This consideration is essential for ensuring the reliability of TRNGs in long-running systems.

D. Artificial Phase-Noise Against Eavesdropping Attacks

1) Overview: In wireless communications, the broadcast nature of signals makes them susceptible to eavesdropping. Conventional encryption methods can be computationally intensive and energy-consuming, which is not ideal for resource-constrained devices. Artificial phase noise offers an innovative solution by leveraging phase noise to obscure

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Algorithm 4: Detailed Steps of the Artificial Phase Noise Scheme Against Eavesdropping [15]

Artificial Phase Noise Generation:
Step 1: Initialize the LFSR to produce a pseudorandom sequence.
Step 2: Use the sequence as a pointer to the LUT to determine the phase shift.
Step 3: Apply the phase shift to the M-QAM symbols before transmission.
Artificial Phase Noise Compensation:
Step 1: At the receiver, use a synchronized LFSR and LUT to determine and compensate for the phase shift.
Step 2: Ensure both LFSRs are synchronized and reset periodically to maintain alignment.

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the transmitted signal, thereby protecting it from unauthorized interception. Artificial phase noise is generated at the transmitter and compensated at the receiver without any feedback link, utilizing simple hardware components such as Linear Feedback Shift Registers (LFSRs) and Look-Up Tables (LUTs).

2) Artificial Phase-Noise Scheme Against Eavesdropping: In the realm of securing wireless communications against eavesdropping, Khan et al. pioneered the introduction of artificial phase noise to protect transmitted signals from unauthorized interception [15]. Their system employs a basic phase noise generator at the transmitter, which is then compensated at the legitimate receiver using synchronized hardware components. This technique ensures that any eavesdropper intercepting the signal experiences a significant degradation in signal quality due to the unknown phase noise. They implemented their method using Software-Defined Radio (SDR) hardware, demonstrating its practicality and effectiveness in real-world scenarios. The results showed that the error probability for the legitimate receiver remains unaffected while the eavesdropper's error probability increases significantly with the introduction of artificial phase noise, particularly beyond certain phase noise levels. This approach offers a low-complexity, energy-efficient alternative to traditional cryptographic methods, making it suitable for applications in resource-constrained environments.

We use the artificial phase noise scheme from [15] as an example to demonstrate the basic steps of an artificial phase noise scheme, as presented in **Algorithm 4**.

3) Lessons: By reviewing the artificial phase noise scheme, we draw the following lessons:

- The method requires minimal computational resources and energy, making it suitable for resource-constrained environments.
- Ensuring precise synchronization between the transmitter and receiver's LFSRs is crucial.
- Maintaining the secrecy of the initial conditions used for the LFSRs is essential to prevent eavesdroppers from predicting the phase noise.

E. Summary

This section provides a comprehensive examination of the diverse applications of phase noise in wireless communications, summarized in Table VII. The exploration begins with how phase noise can be utilized for PLA, significantly enhancing security in wireless networks. We then delve into its

role in transmitter identification. Furthermore, we discuss the utility of phase noise in generating random numbers and its application in protecting transmitted signals from eavesdropping. These applications highlight the dual nature of phase noise, underscoring its role not only as a challenge but also as a valuable resource for advancing wireless communication technologies. This survey emphasizes the urgent need to further explore and leverage phase noise, given its pivotal role in enhancing and securing wireless communications.

VII. FUTURE RESEARCH DIRECTIONS AND CHALLENGES

In this section, we outline an expanded set of research directions, challenges, and potential solutions that could shape the future of phase noise research in wireless communications.

A. Advanced Modulation Techniques for High-Frequency Bands

Focus on developing and analyzing advanced modulation techniques optimized for high-frequency bands, such as millimeter wave and THz frequencies. This includes studying the behavior of phase noise in these bands and creating modulation schemes less susceptible to phase noise effects. Explore the use of non-orthogonal waveforms, new constellation designs, and error-correction techniques that can operate effectively under high phase noise conditions.

Challenge: High-frequency operation exacerbates the impact of phase noise due to wider bandwidths and higher oscillator frequencies.

Potential Solution: To address this challenge, we think it is worthwhile to pursue research along the following directions:

- *Robust Modulation Schemes:* Develop new modulation schemes inherently resistant to phase noise, such as differential phase-shift keying or continuous phase modulation, known for their robustness against phase variations.
- *Oscillator Design Improvements:* Research new materials and technologies to design oscillators with reduced phase noise characteristics at high frequencies, such as using micro-electro-mechanical systems (MEMS) based oscillators or advanced frequency synthesizers.
- *Adaptive Coding Techniques:* Implement adaptive coding techniques that dynamically adjust the coding rate and scheme based on the detected phase noise level to maintain communication reliability.

TABLE VII
DIVERSE APPLICATIONS OF PHASE NOISE IN WIRELESS COMMUNICATIONS

Authors	Year	Applications	Contributions	Open Issues
Sunar *et al.* [117]	2007	Random number generation	A TRNG with enhanced security against active attacks	Increased complexity and cost
Liu *et al.* [118]	2016	Random number generation	A low-cost, efficient TRNG suitable for smart cards	Limited output rate and large size
Zhao *et al.* [12]	2017	PLA	A PLA scheme based on both phase noise and clock	Vulnerable to replay attacks
Cutitta and Dietlein [13]	2018	Transmitter identification	A phase noise measurement scheme to differentiate transmitters based on unique noise characteristics	Requires accurate measurements and long-term stability
Zhang *et al.* [11]	2020	PLA	A PLA scheme based on both channel response and phase noise in a MIMO system	High computational overhead and high latency
Zhang *et al.* [116]	2021	PLA	A PLA scheme based on both channel response and phase noise in an uplink massive MIMO system	High computational overhead and high latency
Xie *et al.* [10]	2022	PLA	Two PLA schemes based on phase noise in a MIMO system	Vulnerable to replay attacks
Khan *et al.* [15]	2022	Eavesdropping resistance	An artificial phase noise scheme implemented using SDR hardware to protect transmitted signals from eavesdropping	Requires precise synchronization and initial condition secrecy
Peetermans and Verbauwhede [14]	2024	Random number generation	A detailed analysis of noise impacts on ASIC TRNGs	Further integration into practical applications remains a challenge

B. Machine Learning for Phase Noise Management

Implement machine learning algorithms to predict, analyze, and mitigate phase noise in real-time. Explore the use of supervised, unsupervised, and reinforcement learning techniques to model the stochastic nature of phase noise and develop predictive models that dynamically adapt to varying conditions. Investigate the integration of machine learning with traditional signal processing to create hybrid systems that enhance phase noise mitigation.

Challenge: Traditional algorithms struggle with the dynamic nature of wireless channels, where phase noise characteristics can vary rapidly.

Potential Solution: To address this challenge, we think it is worthwhile to pursue research along the following directions:

- *Deep Learning Models:* Utilize deep learning models to learn from large datasets and provide real-time phase noise prediction and compensation. Models such as long short-term memory (LSTM) or gated recurrent units (GRU) can capture temporal dependencies in phase noise patterns.
- *Reinforcement Learning for Adaptive Control:* Apply reinforcement learning to develop adaptive control strategies for phase noise mitigation. This approach can continuously optimize system parameters based on real-time feedback from the communication environment.
- *Hybrid Approaches:* Combine machine learning with classical estimation techniques, e.g., Kalman filters, to leverage the strengths of both methods to seek more accurate and robust phase noise management.

C. Quantum Communications

Investigate the specific impacts of phase noise on quantum communication systems, particularly how it affects quantum bit error rates and the overall stability of quantum states. Develop theoretical models and simulation tools to understand the interaction between quantum states and phase noise.

Explore mitigation techniques that can be integrated into quantum key distribution systems and quantum repeaters.

Challenge: Quantum systems are extremely sensitive to phase noise, which can lead to rapid decoherence and loss of information.

Potential Solution: To address this challenge, we think it is worthwhile to pursue research along the following directions:

- *Quantum Error Correction:* Implement quantum error correction codes that can detect and correct phase noise-induced errors. Codes such as Shor code or surface codes can be tailored for phase noise resilience.
- *Advanced Synchronization Techniques:* Develop advanced synchronization techniques to maintain coherence in quantum systems, such as using entangled states for synchronization or employing quantum phase estimation algorithms.
- *Noise Filtering Mechanisms:* Design noise filtering mechanisms specifically for quantum communications, utilizing techniques like dynamic decoupling to isolate the quantum states from phase noise.

D. Cross-Layer Design Optimization

Develop cross-layer optimization strategies that incorporate phase noise considerations into both hardware design and network protocol development. This involves creating models to simulate the cumulative effects of phase noise across different layers and designing algorithms that optimize performance based on these models. Research should also focus on adaptive protocols that can react to changes in phase noise conditions in real time.

Challenge: Ensuring that phase noise mitigation is effective across multiple layers of communication protocols without introducing excessive overhead.

Potential Solution: To address this challenge, we think it is worthwhile to pursue research along the following directions:

- *Integrated Frameworks:* Propose integrated frameworks that combine physical layer mitigation techniques with

network layer adjustments to optimize throughput and reliability.

- *Real-time Adaptive Protocols:* Design adaptive protocols that can adjust parameters such as power control, modulation schemes, and error correction codes in real time based on phase noise conditions.
- *Collaborative Signal Processing:* Implement collaborative signal processing techniques where multiple network nodes work together to detect and mitigate phase noise, sharing information to improve overall network performance.

E. Scalability in Massive MIMO Systems

Address the computational challenges of scaling phase noise compensation techniques in massive MIMO systems. This includes developing decentralized or distributed algorithms for phase noise estimation and compensation that can operate efficiently in systems with a large number of antennas. Research should also explore the potential of using advanced signal processing techniques like machine learning and big data analytics to manage scalability issues.

Challenge: The computational complexity and resource requirements for phase noise compensation scale significantly with the number of antennas.

Potential Solution: To address this challenge, we think it is worthwhile to pursue research along the following directions:

- *Sparse Signal Processing:* Use sparse signal processing techniques to reduce the dimensionality of the problem, making it feasible to handle large antenna arrays without excessive computational overhead.
- *Distributed Algorithms:* Develop distributed algorithms that perform phase noise estimation and compensation in a decentralized manner, leveraging local processing at each antenna or subarray.
- *Big Data Analytics:* Employ big data analytics to process and analyze the vast amounts of data generated by massive MIMO systems, using techniques like principal component analysis to identify and mitigate phase noise patterns efficiently.

F. Secure Communication Protocols

Develop cryptographic protocols that utilize phase noise characteristics as an entropy source to enhance the security of wireless communications. This involves characterizing the randomness of phase noise and integrating it into key generation and distribution processes. Research should also address the challenge of quantifying the security level provided by phase noise and ensuring it can withstand various attack vectors.

Challenge: Phase noise could potentially be exploited by adversaries to breach communication security.

Potential Solution: To address this challenge, we think it is worthwhile to pursue research along the following directions:

- *Entropy-Based Key Generation:* Integrate phase noise into the key generation process, using its inherent randomness to create highly secure cryptographic keys. Techniques like TRNGs can be based on phase noise.

- *Phase Noise Masking:* Develop phase noise masking techniques that obscure communication signals from eavesdroppers, using controlled phase noise to add a layer of security.
- *Security Analysis Frameworks:* Create frameworks to analyze and quantify the security level provided by phase noise-based encryption, ensuring that it meets the required standards and is resistant to known attack methods.

VIII. Conclusion

In this paper, we presented a comprehensive taxonomy for phase noise in terms of phase-noise models, phase-noise estimation and data reception schemes, joint transmit and receive designs with phase-noise mitigation, and applications of phase noise in wireless communications. We divided existing phase-noise models into four categories: phase-noise models associated with an FR oscillator, phase-noise models associated with a PLL circuit, phase-noise models associated with a channel estimator, and presented corresponding performance analyses of various phase-noise models. We discussed three types of phase-noise estimation and data reception schemes: data reception schemes based on classical, Bayesian, and machine-learning based estimation algorithms. We also examined two types of joint transmit and receive designs, with or without a constraint on the shape of the redesigned modulation constellation, to improve phase-noise mitigation performance. We reviewed four applications of phase noise in wireless communications. We meticulously analyze the contributions and identify the limitations and issues of each work reviewed on phase noise. We also provide enlightening lessons and summaries that encapsulate key findings and observations from the research. Moreover, we presented some research directions and accompanying challenges and potential solutions related to phase noise.

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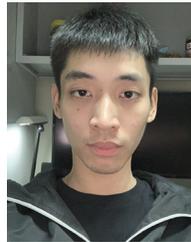
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